

Review Packet for Exam #2

Math 121-D. Benedetto

Integrals: Compute each of the following integrals, or else show that it diverges.

$$1. \int_3^\infty \frac{1}{x^2 - 4x + 7} dx$$

$$2. \int_e^\infty \frac{1}{x(\ln x)^3} dx$$

$$3. \int_0^\infty \frac{1}{(x+2)(2x+5)} dx$$

$$4. \int_7^\infty \frac{1}{x^2 - 8x + 19} dx$$

$$5. \int_2^\infty \frac{1}{x^2 - 2x + 4} dx$$

$$6. \int \frac{1}{x^2 + 2x + 2} dx$$

$$7. \int_0^{\frac{\pi}{2}} \tan x dx$$

$$8. \int_3^4 \frac{1}{(x-4)^2} dx$$

$$9. \int_1^2 \frac{1}{x \ln x} dx$$

$$10. \int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$11. \int_1^\infty \frac{\ln x}{x} dx$$

$$12. \int \frac{2x-5}{x^2 + 2x + 2} dx$$

$$13. \int_0^1 \frac{e^x}{\sqrt{e^x - 1}} dx$$

$$14. \int_0^1 \ln x dx$$

$$15. \int_0^1 \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$$

$$16. \int_{-\infty}^\infty \frac{1}{x^2 - 6x + 10} dx$$

$$17. \int_{-5}^0 \frac{x}{x^2 + 4x - 5} dx$$

$$18. \int \frac{x^5 + 2}{x^2 - 1} dx$$

$$19. \int_0^6 \frac{1}{(x-2)^2} dx$$

$$20. \int_0^\infty \frac{1}{x^2 + 3x + 2} dx$$

$$21. \int_0^{\frac{\pi}{2}} \tan^2 x dx$$

$$22. \int_0^2 \frac{1}{(4-x^2)^{\frac{3}{2}}} dx$$

$$23. \int \frac{4x^2 + 7x + 6}{(x+2)(x^2+4)} dx$$

$$24. \int_1^\infty \frac{1}{x(x+1)} dx$$

$$25. \int_{-3}^{-2} \frac{1}{x^2 - 4} dx$$

$$26. \int_0^1 \arcsin x dx \text{ (leads to improper integral)}$$

$$27. \int \frac{x^3 + 7x + 1}{x^2 + 1} dx$$

$$28. \int \frac{x^4 + x^3 + 2x^2 + 6x + 2}{(x+1)(x^2+1)} dx$$

$$29. \int_0^1 \frac{e^{\frac{1}{x}}}{x^2} dx$$

$$30. \int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^2} dx$$

$$31. \int_{-\infty}^{\infty} \frac{x^2}{9+x^6} dx$$

$$32. \int_2^{\infty} \frac{x}{e^{3x}} dx$$

$$33. \int_0^e \frac{\ln x}{\sqrt{x}} dx$$

Sequences: For each of the following sequences, decide whether it converges or diverges. If it converges, compute its limit.

34. $\left\{ \frac{1+n-7n^4}{3n^4+8n^3+9} \right\}_{n=1}^{\infty}$

35. $\left\{ \frac{n^3}{(n+1)^3} \right\}_{n=1}^{\infty}$

36. $\left\{ \left(\frac{n-5}{n} \right)^n \right\}_{n=1}^{\infty}$

37. $\left\{ \frac{(2n+3)!}{(2n+5)!} \right\}_{n=1}^{\infty}$

38. $\left\{ \arctan(n^2+1) \right\}_{n=1}^{\infty}$

39. $\left\{ \frac{\sqrt{n}}{(\ln n)^2} \right\}_{n=1}^{\infty}$

40. $\left\{ n^{\frac{1}{n}} \right\}_{n=1}^{\infty}$

41. $\left\{ n \sin \left(\frac{1}{n} \right) \right\}_{n=1}^{\infty}$

42. $\left\{ \left(\frac{n+1}{n} \right)^n \right\}_{n=1}^{\infty}$

43. $\left\{ \left(\frac{n}{n+1} \right)^n \right\}_{n=1}^{\infty}$

Series: Find the **sum** for each of the following series (all of which converge):

44. $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}$

45. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{n-1}}{3^{n+1}}$

46. $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n+2}}{2^{4n-1}}$

47. $\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{3^{2n-1}}$

More Series: Determine whether each of the following series **converge** or **diverge**. Name any convergence test(s) you use, and justify all of your work

48.
$$\sum_{n=1}^{\infty} \frac{2n^3 - \ln n}{5n^3 + 9}$$

49.
$$\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

50.
$$\sum_{n=1}^{\infty} \frac{\sqrt{n} + 3}{4n^2 - 2}$$

51.
$$\sum_{n=1}^{\infty} \frac{n^{19} + 40n^6 + 4n^3 + 19}{4 + 17n^5 + n^{20}}$$

52.
$$\sum_{n=2}^{\infty} \frac{e^n}{\ln n}$$

53.
$$\sum_{n=1}^{\infty} \frac{5}{n^5} + \frac{1}{5^n}$$

54.
$$\sum_{n=1}^{\infty} \frac{1 + 3n^3}{n^5}$$

55.
$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^7}$$

56.
$$\sum_{n=1}^{\infty} \frac{\arctan n}{1 + n^2}$$

57.
$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{\arctan n}$$

58.
$$\sum_{n=1}^{\infty} \frac{2n + 5}{5n^3 + 3n^2}$$

59.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 + 5n - 3}$$

60.
$$\sum_{n=1}^{\infty} \frac{\pi}{\arctan(2n)}$$

61.
$$\sum_{n=1}^{\infty} 3 + \frac{1}{3^n}$$

62.
$$\sum_{n=1}^{\infty} e^{\frac{1}{n}}$$

$$63. \sum_{n=1}^{\infty} \frac{6}{n^6} + \frac{1}{(n+1)^6}$$

$$64. \sum_{n=1}^{\infty} \cos^2 \left(\frac{\pi n^2 + n}{n^2 + 7} \right)$$

$$65. \sum_{n=1}^{\infty} (-1)^n \frac{\cos^2(\pi n^2 + 1)}{n^2 + 7}$$

Even More Series: Determine whether each of the following series **converges absolutely**, **converges conditionally**, or **diverges**. Name any convergence test(s) you use, and justify all of your work.

$$66. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{5n+2}$$

$$67. \sum_{n=1}^{\infty} (-1)^n \frac{n^3 + 6n}{n^8 + 1}$$

$$68. \sum_{n=1}^{\infty} \frac{5^{2n}}{(2n+1)! \ln n}$$

$$69. \sum_{n=1}^{\infty} \frac{n! n^6 n^n}{10^{4n} e^{2n}}$$

$$70. \sum_{n=1}^{\infty} (-1)^n \frac{1}{4n+3}$$

$$71. \sum_{n=1}^{\infty} \frac{(n!)^3 e^{2n}}{(3n)! n^n}$$

$$72. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\arctan n}{n^7 + n}$$

$$73. \sum_{n=1}^{\infty} \frac{(-1)^n (3n)! n^2}{8^n (n!)^2 n^n}$$

$$74. \sum_{n=1}^{\infty} (-1)^n \frac{n^3 + 7}{n^7 + 3}$$

$$75. \sum_{n=1}^{\infty} \frac{(-1)^n (\ln n) \pi^n (2n)!}{n^n 4^n n!}$$