Quiz #3 Final Answers — Spring 2021

1. $\int_{-1}^{5} \frac{7}{x^2-3x-10}\,dx = \int_{-1}^{5} \frac{7}{(x-5)(x+2)}\,dx$ ← improper

$= \lim_{t\to 5^-} \int_{-1}^{t} \frac{7}{(x-5)(x+2)}\,dx$

PFD

$\frac{7}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2}$ $\quad (x-5)(x+2)$

$7 = A(x+2) + B(x-5)$
$= Ax + 2A + Bx - 5B$
$= (A+B)x + (2A-5B)$

- $A+B=0 \Rightarrow B=-A$
- $2A-5B=7$

$2A-5(-A)=7$
$7A=7$
$A=1 \Rightarrow B=-1$

$= \lim_{t\to 5^-} \int_{-1}^{t} \frac{1}{x-5} - \frac{1}{x+2}\,dx$

$= \lim_{t\to 5^-} \ln|x-5| - \ln|x+2| \Big|_{-1}^{t}$

$= \lim_{t\to 5^-} \ln|t-5| - \ln|t+2| - [\ln 6 - \ln 1]$

$t \to 5^-$ ⇒ $|t-5| \to 0^+$ ⇒ $\ln|t-5| \to -\infty$ (Diverges)
$\ln|t+2| \to \ln 7$ (Finite)
$\ln 6 - \ln 1$ (Finite)

$= -\infty$ Diverges

2. $\int_{-\infty}^{4} \frac{8}{x^2-4x+8}\,dx = \lim_{t\to-\infty} \int_{t}^{4} \frac{8}{x^2-4x+8}\,dx$

Improper

$= \lim_{t\to-\infty} 8\int_{t}^{4} \frac{1}{(x-2)^2+4}\,dx$ $\quad x^2-4x+4$

$u = x-2$
$du = dx$

$= \lim_{t\to-\infty} 8\int_{t-2}^{2} \frac{1}{u^2+4}\,du$ a-rule

$x = t \Rightarrow u = t-2$
$x = 4 \Rightarrow u = 4-2 = 2$

$= \lim_{t\to-\infty} 8\left(\frac{1}{2}\right)\arctan\left(\frac{u}{2}\right)\Big|_{t-2}^{2}$

$= \lim_{t\to-\infty} 4\left[\arctan\left(\frac{2}{2}\right) - \arctan\left(\frac{t-2}{2}\right)\right]$

$\arctan\left(\frac{2}{2}\right) \to \frac{\pi}{4}$ $\quad \arctan\left(\frac{t-2}{2}\right) \to -\frac{\pi}{2}$ ($t-2 \to -\infty$)

$= 4\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = 4\left(\frac{3\pi}{4}\right) = \boxed{3\pi}$

3. $\int_0^1 x^4 \cdot \ln x \, dx = \lim_{t \to 0^+} \int_t^1 x^4 \cdot \ln x \, dx = \lim_{t \to 0^+} \frac{x^5}{5} \ln x \Big|_t^1 - \frac{1}{5} \int_t^1 x^4 dx$

0 ← Improper

IBP

IBP

$u = \ln x$	$dv = x^4 dx$
$du = \frac{1}{x} dx$	$v = \frac{x^5}{5}$

$= \lim_{t \to 0^+} \frac{x^5}{5} \ln x \Big|_t^1 - \frac{x^5}{25} \Big|_t^1$

$= \lim_{t \to 0^+} \frac{1}{5} \cdot \ln 1 - \frac{t^5}{5} \ln t - \left(\frac{1}{25} - \frac{t^5}{25} \right) = \boxed{-\frac{1}{25}}$

(0 → 0; see (*))

(*) $\lim_{t \to 0^+} t^5 \cdot \ln t$ $(0 \cdot (-\infty))$ $= \lim_{t \to 0^+} \frac{\ln t}{\frac{1}{t^5}}$ $\left(\frac{-\infty}{\infty}\right)$

$= \lim_{t \to 0^+} \frac{\frac{1}{t}}{\frac{-5}{t^6}}$ $\left(\frac{-t^6}{5}\right)$

$= \lim_{t \to 0^+} \frac{-t^5}{5} = 0$ (0)