

# Quiz #2 Final Answers Spring 2021

$$1. \lim_{x \rightarrow 0} \frac{\ln(1-5x) + \arcsin(5x)}{3xe^x - \arctan(3x)}$$

prep

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{1-5x}(-5) + \frac{1}{\sqrt{1-(5x)^2}} \cdot 5}{3xe^x + 3e^x - \frac{1}{1+(3x)^2} \cdot 3}$$

prep

$$= \lim_{x \rightarrow 0} \frac{5(1-5x)^{-2}(-5) - \frac{5}{2}(1-25x^2)^{-3/2}(-50x)}{3xe^x + 3e^x + 3e^x + 3(1+9x^2)^{-2}(18x)}$$

rewrite

$$= \lim_{x \rightarrow 0} \frac{\frac{-25}{(1-5x)^2} + \frac{250x}{2(1-25x^2)^{3/2}}}{3xe^x + 6e^x + \frac{54x}{(1+9x^2)^2}}$$

$$= \boxed{-\frac{25}{6}} \quad \text{Match!}$$

$$2. \lim_{x \rightarrow \infty} \left(1 - \frac{8}{x^3}\right)^{x^3} = e^{\lim_{x \rightarrow \infty} \ln \left[ \left(1 - \frac{8}{x^3}\right)^{x^3} \right]}$$

$$= e^{\lim_{x \rightarrow \infty} x^3 \ln \left(1 - \frac{8}{x^3}\right)}$$

$\infty \cdot 0$   
FLIP

$$= e^{\lim_{x \rightarrow \infty} \frac{\ln \left(1 - \frac{8}{x^3}\right)}{\frac{1}{x^3}}}$$

scratch  
 $-8x^{-3} \rightarrow 24x^{-4}$   
 $\frac{0}{0}$   
 $\frac{1}{x^3} \rightarrow x^{-3} \rightarrow -3x^{-4}$

$$\stackrel{L'H}{=} e^{\lim_{x \rightarrow \infty} \frac{\frac{1}{1 - \frac{8}{x^3}} \left(\frac{24}{x^4}\right)}{\frac{-3}{x^4}}}$$

FLIP  
BACK  
ALGEBRA

$$= e^{\lim_{x \rightarrow \infty} \frac{1}{1 - \frac{8}{x^3}} \cdot (-8)}$$

don't drop

$$= e^{-8}$$

Match!

3.  $\int_0^1 (x+1) \arctan x \, dx = \left(\frac{x^2}{2} + x\right) \arctan x \Big|_0^1 - \int_0^1 \frac{\frac{x^2}{2} + x}{1+x^2} dx$

FBP distribute split-split

$u = \arctan x \quad dv = x+1 \, dx$   
 $du = \frac{1}{1+x^2} \quad v = \frac{x^2}{2} + x$

$= \left(\frac{x^2}{2} + x\right) \arctan x \Big|_0^1 - \frac{1}{2} \int_0^1 \frac{x^2 + 1 - 1}{1+x^2} dx - \int_0^1 \frac{x}{1+x^2} dx$

slip-in/slip-out u-sub.

$= \left(\frac{x^2}{2} + x\right) \arctan x \Big|_0^1 - \frac{1}{2} \left[ \int_0^1 \frac{x^2 + 1}{1+x^2} dx - \int_0^1 \frac{1}{1+x^2} dx \right] - \frac{1}{2} \int_1^2 \frac{1}{u} du$

$u = 1+x^2$   
 $du = 2x \, dx$   
 $\frac{1}{2} du = x \, dx$

$= \left(\frac{x^2}{2} + x\right) \arctan x \Big|_0^1 - \frac{1}{2} \left[ x - \arctan x \right] \Big|_0^1 - \frac{1}{2} \ln|u| \Big|_1^2$

$= \frac{3}{2} \arctan 1 - 0 - \frac{1}{2} \left[ 1 - \arctan 1 - (0-0) \right] - \frac{1}{2} (\ln 2 - \ln 1)$

$\nearrow \pi/4$   $\curvearrowright$   $\nearrow \pi/4$   $\curvearrowright$

$x=0 \Rightarrow u=1$   
 $x=1 \Rightarrow u=2$

$= \frac{3\pi}{8} - \frac{1}{2} + \frac{\pi}{8} - \frac{\ln 2}{2}$

$= \frac{\cancel{4\pi}}{8} - \frac{1}{2} - \frac{\ln 2}{2} = \frac{\pi - 1 - \ln 2}{2}$

Match!

OR: You can split integral into 2 separate pieces.

$\int_0^1 (x+1) \arctan x \, dx = \int_0^1 \underbrace{x \arctan x}_{\text{FBP 1}} + \underbrace{\arctan x}_{\text{FBP 2}} \, dx \dots$

Optional Bonus:

$$\int \ln(3x^2+4) \overset{1}{dx} = x \ln(3x^2+4) - \overset{2 \cdot 3}{6} \int \frac{x^2}{3x^2+4} dx$$

pull in 3  
find match?

$$u = \ln(3x^2+4) \quad dv = 1 dx$$
$$du = \frac{1}{3x^2+4} (6x) \quad v = x$$

$$= x \ln(3x^2+4) - 2 \int \frac{3x^2+4-4}{3x^2+4} dx$$

Keep pulling out constants  
Slip-in / Slip-out  
split-split

$$= x \ln(3x^2+4) - 2 \left[ \int \frac{3x^2+4}{3x^2+4} dx - 4 \int \frac{1}{3x^2+4} dx \right]$$

$$= x \ln(3x^2+4) - 2 \left[ x - 4 \int \frac{1}{(\sqrt{3}x)^2+4} dx \right]$$

u-sub here  
prep for "a"-rule

$$= x \ln(3x^2+4) - 2x + \frac{8}{\sqrt{3}} \int \frac{1}{u^2+4} du$$

$$u = \sqrt{3}x$$
$$du = \sqrt{3} dx$$
$$\frac{1}{\sqrt{3}} du = dx$$

$$= x \ln(3x^2+4) - 2x + \frac{4}{\sqrt{3}} \cdot \frac{1}{2} \arctan\left(\frac{u}{2}\right) + C$$

now ready for "a"-rule"

$$= x \ln(3x^2+4) - 2x + \frac{4}{\sqrt{3}} \arctan\left(\frac{\sqrt{3}x}{2}\right) + C$$