Instructions:

• This is an Open Notes Quiz. You can use materials, homeworks problems, lecture notes, etc. that you manually worked on.

• This is **NOT** an Open Internet Quiz. You can only access our Main Course Webpage.

• You are not allowed to work on or discuss these problems with other students, professor, Math Fellow TA or simply put anyone.

• You can ask a few small, clarifying, questions in Office Hours, but the problems will not be solved for you.

• The main goal is to make a thoughtful and detailed presentation for the solutions. Submit a clear final draft. No mess please.

• Please submit your final work in Gradescope in the Quiz 1 entry.

1. [10 Points]

(a) Let $y = \arctan x$. Use differentiation to **PROVE** that $\dfrac{dy}{dx} = \dfrac{1}{1 + x^2}$.

(b) From part (a) we now know that $\displaystyle\int \dfrac{1}{1 + x^2}\, dx = \arctan x + C$. You may use this fact to **PROVE** that

$$\int \frac{1}{25 + x^2}\, dx = \frac{1}{5} \arctan\left(\frac{x}{5}\right) + C \quad \longleftarrow \textbf{ Prove this.}$$

2. [10 Points] Show that $\displaystyle\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16 - x^2}} + \frac{1}{4 + x^2}\, dx = \boxed{\dfrac{5\pi}{24}}$

3. [10 Points] Show that $\displaystyle\int_{-\ln 2}^{-\ln\left(\frac{2}{\sqrt{3}}\right)} \frac{e^x}{\sqrt{1 - e^{2x}}}\, dx = \boxed{\dfrac{\pi}{6}}$

4. [10 Points] Compute $\displaystyle\int \frac{x^2}{x^2 + 3}\, dx$