

- Please see the course webpage for the answer key.

1. (a) Use the MacLaurin Series representation for $f(x) = x^3 \sin(x^2)$ to

$$\text{Estimate } \int_0^1 x^3 \sin(x^2) dx \text{ with error less than } \frac{1}{100}.$$

Justify in words that your error is indeed less than $\frac{1}{100}$.

(b) Estimate $\frac{1}{\sqrt{e}}$ with error less than $\frac{1}{100}$. Justify.

2. Find the **sum** for each of the following series.

(a) $1 - 2 + \frac{4}{2!} - \frac{8}{3!} + \frac{16}{4!} - \frac{32}{5!} + \dots$

(b) $\frac{1}{3\pi} - \frac{1}{18\pi^2} + \frac{1}{81\pi^3} - \frac{1}{324\pi^4} + \dots$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n (\ln 9)^n}{2^{n+1} n!}$

(d) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$

(e) $-\frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \frac{\pi^8}{8!} - \dots$

(f) $\frac{1}{\sqrt{3}} - \frac{1}{9\sqrt{3}} + \frac{1}{45\sqrt{3}} - \frac{1}{189\sqrt{3}} + \dots$

(g) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{9^n (2n+1)!}$

(h) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{2^{4n} (2n)!}$