Name: Answer Key

Math 121 Self-Assessment Quiz #10 November 20, 2019

- Please see the course webpage for the answer key.

1. (a) Write the MacLaurin Series for $f(x) = x^4 \arctan(2x)$. State the Radius of Convergence.

$$x^4 \arctan(2x) = x^4 \sum_{n=1}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{2n+1} = x^4 \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+5}}{2n+1}$$

$$= \frac{2x^5}{1} \boxed{- \frac{2^3 x^7}{3}} + \boxed{\frac{2^5 x^9}{5}} - \cdots$$

$\boxed{0} x^8$

Need $|2x| < 1 \Rightarrow |x| < \frac{1}{2}$

$\Rightarrow \boxed{R = \frac{1}{2}}$

(b) Use this Series to determine the **seventh**, **eighth**, and **ninth** derivatives of $f(x) = x^4 \arctan(2x)$ evaluated at $x = 0$. Do NOT simplify your answer this time.

Maclaurin Series

$$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \boxed{\frac{f^{(7)}(0)}{7!}} x^7 + \boxed{\frac{f^{(8)}(0)}{8!}} x^8 + \boxed{\frac{f^{(9)}(0)}{9!}} x^9 + \cdots$$

Equate Coefficients

$$\frac{f^{(7)}(0)}{7!} = \frac{-8}{3} \Rightarrow f^{(7)}(0) = \frac{-8 \cdot 7!}{3} = \boxed{\frac{-8!}{3}}$$

$$\frac{f^{(8)}(0)}{8!} = 0 \quad \Rightarrow f^{(8)}(0) = 0$$

(no x^8 term)

$$\frac{f^{(9)}(0)}{9!} = \frac{32}{5} \Rightarrow f^{(9)}(0) = \boxed{\frac{32 \cdot 9!}{5}}$$

Ratio Test **2.** (a) Use the Infinite Series $\sum_{n=1}^{\infty} \frac{4^n}{n!}$ to compute $\lim_{n\to\infty} \frac{4^n}{n!} =$.

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\frac{4^{n+1}}{(n+1)!}}{\frac{4^n}{n!}}\right| = \lim_{n\to\infty} \left(\frac{4^{n+1}}{4^n} \cdot \frac{n!}{(n+1)!}\right) = \lim_{n\to\infty} \frac{4}{\underbrace{n+1}_{\infty}} = 0 < 1$$ Series Converges by RT

$\Rightarrow \lim_{n\to\infty} \frac{4^n}{n!} = \boxed{0}$ because otherwise, if $\lim_{n\to\infty} \frac{4^n}{n!} \neq 0$, then the series would Diverge by nTDT, which would contradict the proof above

(b) Use the Infinite Series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ to compute $\lim_{n\to\infty} \frac{n!}{n^n} =$.

Ratio Test

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}}\right| = \lim_{n\to\infty} \frac{\overbrace{(n+1)!}^{\cancel{(n+1)}\,\cancel{n!}}}{\cancel{n!}} \cdot \frac{n^n}{\underbrace{(n+1)^{n+1}}_{(n+1)^n \cancel{(n+1)}}} = \lim_{n\to\infty} \frac{n^n}{(n+1)^n} = \frac{1}{e} < 1$$ Series Converges by R.T

$\Rightarrow \lim_{n\to\infty} \frac{n!}{n^n} = \boxed{0}$ because otherwise if $\lim_{n\to\infty} \frac{n!}{n^n} \neq 0$, then the series would Diverge by nTDT, which would contradict the proof above.