31. $\int_1^5 \frac{M}{e^M}\,dM$

32. $\int_1^2 \frac{(\ln x)^2}{x^3}\,dx$

33. $\int_0^{\pi/3} \sin x \ln(\cos x)\,dx$

34. $\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$

35. $\int_1^2 x^4(\ln x)^2\,dx$

36. $\int_0^t e^s \sin(t-s)\,ds$

37–42 First make a substitution and then use integration by parts to evaluate the integral.

37. $\int e^{\sqrt{x}}\,dx$

38. $\int \cos(\ln x)\,dx$

39. $\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2)\,d\theta$

40. $\int_0^{\pi} e^{\cos t} \sin 2t\,dt$

41. $\int x \ln(1+x)\,dx$

42. $\int \frac{\arcsin(\ln x)}{x}\,dx$

43–46 Evaluate the indefinite integral. Illustrate, and check that your answer is reasonable, by graphing both the function and its antiderivative (take $C = 0$).

43. $\int xe^{-2x}\,dx$

44. $\int x^{3/2} \ln x\,dx$

45. $\int x^3\sqrt{1+x^2}\,dx$

46. $\int x^2 \sin 2x\,dx$

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47. (a) Use the reduction formula in Example 6 to show that

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

(b) Use part (a) and the reduction formula to evaluate $\int \sin^4 x\,dx$.

48. (a) Prove the reduction formula

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x \sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

(b) Use part (a) to evaluate $\int \cos^2 x\,dx$.
(c) Use parts (a) and (b) to evaluate $\int \cos^4 x\,dx$.

49. (a) Use the reduction formula in Example 6 to show that

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{n-1}{\quad}\int_0^{\pi/2} \sin^{n-2}x\,dx$$

50. Prove that, for even powers of sine,

$$\int_0^{\pi/2} \sin^{2n}x\,dx = \frac{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}{2\cdot 4\cdot 6\cdot \cdots \cdot 2n}\,\frac{\pi}{2}$$

51–54 Use integration by parts to prove the reduction formula.

51. $\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1}\,dx$

52. $\int x^n e^x\,dx = x^n e^x - n\int x^{n-1}e^x\,dx$

53. $\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx \quad (n \neq 1)$

54. $\int \sec^n x\,dx = \frac{\tan x \sec^{n-2}x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx \quad (n \neq 1)$

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55. Use Exercise 51 to find $\int (\ln x)^3\,dx$.

56. Use Exercise 52 to find $\int x^4 e^x\,dx$.

57–58 Find the area of the region bounded by the given curves.

57. $y = x^2 \ln x$, $y = 4 \ln x$

58. $y = x^2e^{-x}$, $y = xe^{-x}$

59–60 Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the area of the region bounded by the curves.

59. $y = \arcsin\left(\frac{1}{2}x\right)$, $y = 2 - x^2$

60. $y = x \ln(x+1)$, $y = 3x - x^2$

61–64 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves about the given axis.

61. $y = \cos(\pi x/2)$, $y = 0$, $0 \leq x \leq 1$; about the y-axis

62. $y = e^x$, $y = e^{-x}$, $x = 1$; about the y-axis

63. $y = e^{-x}$, $y = 0$, $x = -1$, $x = 0$; about $x = 1$

64. $y = e^x$, $x = 0$, $y = 3$; about the x-axis

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65. Calculate the volume generated by rotating the region bounded by the curves $y = \ln x$, $y = 0$, and $x = 2$ about each axis.

EXAMPLE 9 Evaluate $\int \sin 4x \cos 5x\, dx$.

SOLUTION This integral could be evaluated using integration by parts, but it's easier to use the identity in Equation 2(a) as follows:

$$\begin{aligned}\int \sin 4x \cos 5x\, dx &= \int \tfrac{1}{2}[\sin(-x) + \sin 9x]\, dx \\ &= \tfrac{1}{2}\int (-\sin x + \sin 9x)\, dx \\ &= \tfrac{1}{2}\left(\cos x - \tfrac{1}{9}\cos 9x\right) + C\end{aligned}$$

7.2 EXERCISES

1–49 Evaluate the integral.

1. $\int \sin^2 x \cos^3 x\, dx$

2. $\int \sin^3\theta \cos^4\theta\, d\theta$

3. $\int_0^{\pi/2} \sin^7\theta \cos^5\theta\, d\theta$

4. $\int_0^{\pi/2} \sin^5 x\, dx$

5. $\int \sin^5(2t) \cos^2(2t)\, dt$

6. $\int t \cos^5(t^2)\, dt$

7. $\int_0^{\pi/2} \cos^2\theta\, d\theta$

8. $\int_0^{2\pi} \sin^2\left(\tfrac{1}{3}\theta\right) d\theta$

9. $\int_0^{\pi} \cos^4(2t)\, dt$

10. $\int_0^{\pi} \sin^2 t \cos^4 t\, dt$

11. $\int_0^{\pi/2} \sin^2 x \cos^2 x\, dx$

12. $\int_0^{\pi/2} (2 - \sin\theta)^2\, d\theta$

13. $\int \sqrt{\cos\theta}\, \sin^3\theta\, d\theta$

14. $\int \dfrac{\sin^2(1/t)}{t^2}\, dt$

15. $\int \cot x \cos^2 x\, dx$

16. $\int \tan^2 x \cos^3 x\, dx$

17. $\int \sin^2 x \sin 2x\, dx$

18. $\int \sin x \cos\left(\tfrac{1}{2}x\right) dx$

19. $\int t \sin^2 t\, dt$

20. $\int x \sin^3 x\, dx$

21. $\int \tan x \sec^3 x\, dx$

22. $\int \tan^2\theta \sec^4\theta\, d\theta$

23. $\int \tan^2 x\, dx$

24. $\int (\tan^2 x + \tan^4 x)\, dx$

25. $\int \tan^4 x \sec^6 x\, dx$

26. $\int_0^{\pi/4} \sec^6\theta \tan^6\theta\, d\theta$

27. $\int \tan^3 x \sec x\, dx$

28. $\int \tan^5 x \sec^3 x\, dx$

29. $\int \tan^3 x \sec^6 x\, dx$

30. $\int_0^{\pi/4} \tan^4 t\, dt$