• $\lim_{x\to 1/2} \frac{6x^2+5x-4}{4x^2+16x-9}$

$\lim_{x\to 0} \frac{\tan 3x}{\sin 2x}$

$\lim_{x\to 0} \frac{x^2}{1-\cos x}$

$\lim_{\theta\to \pi} \frac{1+\cos\theta}{1-\cos\theta}$

$\text{im}_{\to\infty} \frac{x+x^2}{1-2x^2}$

$\text{m}_{\to\infty} \frac{\ln\sqrt{x}}{x^2}$

$\frac{8^t-5^t}{t}$

$\frac{e^{u/10}}{u^3}$

$\frac{\sinh x - x}{x^3}$

$\frac{-\sin x}{-\tan x}$

$\frac{\text{n}\,x)^2}{x}$

$\frac{mx - \cos nx}{x^2}$

$\frac{\text{n}(x-1)}{-x-1}$

$\frac{{}^x - 1}{+x-1}$

$\frac{e^{-x}-2x}{\sin x}$

$\frac{\ln(x-a)}{{}^x - e^a)}$

51. $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

52. $\lim_{x\to 0} (\csc x - \cot x)$

53. $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{e^x-1}\right)$

54. $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\tan^{-1} x}\right)$

55. $\lim_{x\to \infty} (x - \ln x)$

56. $\lim_{x\to 1^+} [\ln(x^7-1) - \ln(x^5-1)]$

57. $\lim_{x\to 0^+} x^{\sqrt{x}}$

58. $\lim_{x\to 0^+} (\tan 2x)^x$

59. $\lim_{x\to 0} (1-2x)^{1/x}$

60. $\lim_{x\to \infty} \left(1+\frac{a}{x}\right)^{bx}$

61. $\lim_{x\to 1^+} x^{1/(1-x)}$

62. $\lim_{x\to \infty} x^{(\ln 2)/(1+\ln x)}$

63. $\lim_{x\to \infty} x^{1/x}$

64. $\lim_{x\to \infty} x^{e^{-x}}$

65. $\lim_{x\to 0^+} (4x+1)^{\cot x}$

66. $\lim_{x\to 1} (2-x)^{\tan(\pi x/2)}$

67. $\lim_{x\to 0^+} (1+\sin 3x)^{1/x}$

68. $\lim_{x\to \infty} \left(\frac{2x-3}{2x+5}\right)^{2x+1}$

69–70 Use a graph to estimate the value of the limit. Then use l'Hospital's Rule to find the exact value.

69. $\lim_{x\to \infty} \left(1+\frac{2}{x}\right)^x$

70. $\lim_{x\to 0} \frac{5^x-4^x}{3^x-2^x}$

71–72 Illustrate l'Hospital's Rule by graphing both $f(x)/g(x)$ and $f'(x)/g'(x)$ near $x = 0$ to see that these ratios have the same limit as $x \to 0$. Also, calculate the exact value of the limit.

71. $f(x) = e^x - 1,\quad g(x) = x^3 + 4x$

72. $f(x) = 2x \sin x,\quad g(x) = \sec x - 1$

73. Prove that

$$\lim_{x\to\infty} \frac{e^x}{x^n} = \infty$$

7.1 EXERCISES

1–2 Evaluate the integral using integration by parts with the indicated choices of u and dv.

1. $\int xe^{2x}\,dx; \quad u = x,\ dv = e^{2x}\,dx$

2. $\int \sqrt{x}\ln x\,dx; \quad u = \ln x,\ dv = \sqrt{x}\,dx$

3–36 Evaluate the integral.

3. $\int x\cos 5x\,dx$

4. $\int ye^{0.2y}\,dy$

5. $\int te^{-3t}\,dt$

6. $\int (x-1)\sin \pi x\,dx$

7. $\int (x^2+2x)\cos x\,dx$

8. $\int t^2\sin \beta t\,dt$

9. $\int \cos^{-1}x\,dx$

10. $\int \ln\sqrt{x}\,dx$

11. $\int t^4\ln t\,dt$

12. $\int \tan^{-1}2y\,dy$

13. $\int t\csc^2 t\,dt$

14. $\int x\cosh ax\,dx$

15. $\int (\ln x)^2\,dx$

16. $\int \frac{z}{10^z}\,dz$

17. $\int e^{2\theta}\sin 3\theta\,d\theta$

18. $\int e^{-\theta}\cos 2\theta\,d\theta$

19. $\int z^3e^z\,dz$

20. $\int x\tan^2 x\,dx$

21. $\int \frac{xe^{2x}}{(1+2x)^2}\,dx$

22. $\int (\arcsin x)^2\,dx$

23. $\int_0^{1/2} x\cos \pi x\,dx$

24. $\int_0^1 (x^2+1)e^{-x}\,dx$

25. $\int_0^2 y\sinh y\,dy$

26. $\int_1^2 w^2\ln w\,dw$

27. $\int_1^5 \frac{\ln R}{R^2}\,dR$

28. $\int_0^{2\pi} t^2\sin 2t\,dt$

29. $\int_0^{\pi} x\sin x\cos x\,dx$

30. $\int_1^{\sqrt{3}} \arctan(1/x)\,dx$

31. $\int_1^5 \frac{M}{e^M}\, dM$

32. $\int_1^2 \frac{(\ln x)^2}{x^3}\, dx$

33. $\int_0^{\pi/3} \sin x \ln(\cos x)\, dx$

34. $\int_0^1 \frac{r^3}{\sqrt{4 + r^2}}\, dr$

35. $\int_1^2 x^4(\ln x)^2\, dx$

36. $\int_0^t e^s \sin(t - s)\, ds$

37–42 First make a substitution and then use integration by parts to evaluate the integral.

37. $\int e^{\sqrt{x}}\, dx$

38. $\int \cos(\ln x)\, dx$

39. $\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2)\, d\theta$

40. $\int_0^{\pi} e^{\cos t} \sin 2t\, dt$

41. $\int x \ln(1 + x)\, dx$

42. $\int \frac{\arcsin(\ln x)}{x}\, dx$

43–46 Evaluate the indefinite integral. Illustrate, and check that your answer is reasonable, by graphing both the function and its antiderivative (take $C = 0$).

43. $\int xe^{-2x}\, dx$

44. $\int x^{3/2} \ln x\, dx$

45. $\int x^3\sqrt{1 + x^2}\, dx$

46. $\int x^2 \sin 2x\, dx$

47. (a) Use the reduction formula in Example 6 to show that

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

(b) Use part (a) and the reduction formula to evaluate $\int \sin^4 x\, dx$.

48. (a) Prove the reduction formula

$$\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1}x \sin x + \frac{n - 1}{n}\int \cos^{n-2}x\, dx$$

(b) Use part (a) to evaluate $\int \cos^2 x\, dx$.
(c) Use parts (a) and (b) to evaluate $\int \cos^4 x\, dx$.

49. (a) Use the reduction formula in Example 6 to show that

$$\int_0^{\pi/2} \sin^n x\, dx = \frac{n - 1}{n}\int_0^{\pi/2} \sin^{n-2}x\, dx$$

where $n \geqslant 2$ is an integer.
(b) Use part (a) to evaluate $\int_0^{\pi/2} \sin^3 x\, dx$ and $\int_0^{\pi/2} \sin^5 x\, dx$.
(c) Use part (a) to show that, for odd powers of sine,

$2 \cdot 4 \cdot 6 \cdot \cdots \cdot 2n$

50. Prove that, for even powers of sine,

$$\int_0^{\pi/2} \sin^{2n}x\, dx = \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n - 1)}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot 2n}\,\frac{\pi}{2}$$

51–54 Use integration by parts to prove the reduction formula.

51. $\int (\ln x)^n\, dx = x(\ln x)^n - n\int (\ln x)^{n-1}\, dx$

52. $\int x^n e^x\, dx = x^n e^x - n\int x^{n-1}e^x\, dx$

53. $\int \tan^n x\, dx = \frac{\tan^{n-1}x}{n - 1} - \int \tan^{n-2}x\, dx \quad (n \neq 1)$

54. $\int \sec^n x\, dx = \frac{\tan x \sec^{n-2}x}{n - 1} + \frac{n - 2}{n - 1}\int \sec^{n-2}x\, dx \quad (n \neq 1)$

55. Use Exercise 51 to find $\int (\ln x)^3\, dx$.

56. Use Exercise 52 to find $\int x^4 e^x\, dx$.

57–58 Find the area of the region bounded by the given curves.

57. $y = x^2 \ln x, \quad y = 4 \ln x$

58. $y = x^2 e^{-x}, \quad y = xe^{-x}$

59–60 Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the area of the region bounded by the curves.

59. $y = \arcsin\left(\tfrac{1}{2}x\right), \quad y = 2 - x^2$

60. $y = x\ln(x + 1), \quad y = 3x - x^2$

61–64 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves about the given axis.

61. $y = \cos(\pi x/2),\ y = 0,\ 0 \leqslant x \leqslant 1;$ about the y-axis

62. $y = e^x,\ y = e^{-x},\ x = 1;$ about the y-axis

63. $y = e^{-x},\ y = 0,\ x = -1,\ x = 0;$ about $x = 1$

64. $y = e^x,\ x = 0,\ y = 3;$ about the x-axis

65. Calculate the volume generated by rotating the region bounded by the curves $y = \ln x$, $y = 0$, and $x = 2$ about each axis.
(a) The y-axis
(b) The x-axis

66. Calculate the average value of $f(x) = x \sec^2 x$ on the interval $[0, \pi/4]$.

67. The Fresnel function $S(x) = \int_0^x \sin\left(\tfrac{1}{2}\pi t^2\right) dt$ was discussed in Example 4.3.3 and is used extensively in the theory of optics. Find $\int S(x)\, dx$. [Your answer will involve $S(x)$.]