

11.  $\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + 1}{x^3 - 1}$
12.  $\lim_{x \rightarrow 1/2} \frac{6x^2 + 5x - 4}{4x^2 + 16x - 9}$
13.  $\lim_{x \rightarrow (\pi/2)^+} \frac{\cos x}{1 - \sin x}$
14.  $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 2x}$
15.  $\lim_{t \rightarrow 0} \frac{e^{2t} - 1}{\sin t}$
16.  $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}$
17.  $\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{1 + \cos 2\theta}$
18.  $\lim_{\theta \rightarrow \pi} \frac{1 + \cos \theta}{1 - \cos \theta}$
19.  $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$
20.  $\lim_{x \rightarrow \infty} \frac{x + x^2}{1 - 2x^2}$
21.  $\lim_{x \rightarrow 0^+} \frac{\ln x}{x}$
22.  $\lim_{x \rightarrow \infty} \frac{\ln \sqrt{x}}{x^2}$
23.  $\lim_{t \rightarrow 1} \frac{t^8 - 1}{t^5 - 1}$
24.  $\lim_{t \rightarrow 0} \frac{8^t - 5^t}{t}$
25.  $\lim_{x \rightarrow 0} \frac{\sqrt{1 + 2x} - \sqrt{1 - 4x}}{x}$
26.  $\lim_{u \rightarrow \infty} \frac{e^{u/10}}{u^3}$
27.  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$
28.  $\lim_{x \rightarrow 0} \frac{\sinh x - x}{x^3}$
29.  $\lim_{x \rightarrow 0} \frac{\tanh x}{\tan x}$
30.  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x - \tan x}$
31.  $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x}$
32.  $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x}$
33.  $\lim_{x \rightarrow 0} \frac{x^{3^x}}{3^x - 1}$
34.  $\lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2}$
35.  $\lim_{x \rightarrow 0} \frac{\ln(1 + x)}{\cos x + e^x - 1}$
36.  $\lim_{x \rightarrow 1} \frac{x \sin(x - 1)}{2x^2 - x - 1}$
37.  $\lim_{x \rightarrow 0^+} \frac{\arctan(2x)}{\ln x}$
38.  $\lim_{x \rightarrow 0^+} \frac{x^x - 1}{\ln x + x - 1}$
39.  $\lim_{x \rightarrow 1} \frac{x^a - 1}{x^b - 1}, b \neq 0$
40.  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$
41.  $\lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$
42.  $\lim_{x \rightarrow a^+} \frac{\cos x \ln(x - a)}{\ln(e^x - e^a)}$
43.  $\lim_{x \rightarrow \infty} x \sin(\pi/x)$
44.  $\lim_{x \rightarrow \infty} \sqrt{x} e^{-x/2}$
45.  $\lim_{x \rightarrow 0} \sin 5x \csc 3x$
46.  $\lim_{x \rightarrow -\infty} x \ln\left(1 - \frac{1}{x}\right)$
47.  $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$
48.  $\lim_{x \rightarrow \infty} x^{3/2} \sin(1/x)$
49.  $\lim_{x \rightarrow 1^+} \ln x \tan(\pi x/2)$
50.  $\lim_{x \rightarrow (\pi/2)^-} \cos x \sec 5x$
51.  $\lim_{x \rightarrow 1} \left( \frac{x}{x-1} - \frac{1}{\ln x} \right)$
52.  $\lim_{x \rightarrow 0} (\csc x - \cot x)$
53.  $\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{e^x - 1} \right)$
54.  $\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{\tan^{-1} x} \right)$
55.  $\lim_{x \rightarrow \infty} (x - \ln x)$
56.  $\lim_{x \rightarrow 1^+} [\ln(x^7 - 1) - \ln(x^5 - 1)]$
57.  $\lim_{x \rightarrow 0^+} x^{\sqrt{x}}$
58.  $\lim_{x \rightarrow 0^+} (\tan 2x)^x$
59.  $\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$
60.  $\lim_{x \rightarrow \infty} \left( 1 + \frac{a}{x} \right)^{bx}$
61.  $\lim_{x \rightarrow 1^+} x^{1/(1-x)}$
62.  $\lim_{x \rightarrow \infty} x^{(\ln 2)/(1 + \ln x)}$
63.  $\lim_{x \rightarrow \infty} x^{1/x}$
64.  $\lim_{x \rightarrow \infty} x^{\epsilon^{-x}}$
65.  $\lim_{x \rightarrow 0^+} (4x + 1)^{\cot x}$
66.  $\lim_{x \rightarrow 1} (2 - x)^{\sin(\pi x/2)}$
67.  $\lim_{x \rightarrow 0^+} (1 + \sin 3x)^{1/x}$
68.  $\lim_{x \rightarrow \infty} \left( \frac{2x - 3}{2x + 5} \right)^{2x+1}$
- 69–70 Use a graph to estimate the value of the limit. Then use l'Hospital's Rule to find the exact value.
69.  $\lim_{x \rightarrow \infty} \left( 1 + \frac{2}{x} \right)^x$
70.  $\lim_{x \rightarrow 0} \frac{5^x - 4^x}{3^x - 2^x}$
- 71–72 Illustrate l'Hospital's Rule by graphing both  $f(x)/g(x)$  and  $f'(x)/g'(x)$  near  $x = 0$  to see that these ratios have the same limit as  $x \rightarrow 0$ . Also, calculate the exact value of the limit.
71.  $f(x) = e^x - 1, g(x) = x^3 + 4x$
72.  $f(x) = 2x \sin x, g(x) = \sec x - 1$
73. Prove that
- $$\lim_{x \rightarrow \infty} \frac{e^x}{x^n} = \infty$$
- for any positive integer  $n$ . This shows that the exponential function approaches infinity faster than any power of  $x$ .
74. Prove that
- $$\lim_{x \rightarrow \infty} \frac{\ln x}{x^p} = 0$$
- for any number  $p > 0$ . This shows that the logarithmic function approaches infinity more slowly than any power of  $x$ .