which diverges by the Test for Divergence [$(-1)^n n$ doesn't converge to 0]. When $x = 1$, the series is

$$\sum_{n=0}^{\infty} \frac{n(3)^n}{3^{n+1}} = \frac{1}{3} \sum_{n=0}^{\infty} n$$

which also diverges by the Test for Divergence. Thus the series converges only when $-5 < x < 1$, so the interval of convergence is $(-5, 1)$.

11.8 EXERCISES

1. What is a power series?

2. (a) What is the radius of convergence of a power series? How do you find it?

(b) What is the interval of convergence of a power series? How do you find it?

3–28 Find the radius of convergence and interval of convergence of the series.

3. $\sum_{n=1}^{\infty} (-1)^n n x^n$

4. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$

5. $\sum_{n=1}^{\infty} \frac{x^n}{2n - 1}$

6. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

7. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

8. $\sum_{n=1}^{\infty} n^n x^n$

9. $\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$

10. $\sum_{n=1}^{\infty} 2^n n^2 x^n$

11. $\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{\sqrt{n}} x^n$

12. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n 5^n} x^n$

13. $\sum_{n=1}^{\infty} \frac{n}{2^n (n^2 + 1)} x^n$

14. $\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$

15. $\sum_{n=0}^{\infty} \frac{(x - 2)^n}{n^2 + 1}$

16. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n - 1)2^n} (x - 1)^n$

17. $\sum_{n=2}^{\infty} \frac{(x + 2)^n}{2^n \ln n}$

18. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n} (x + 6)^n$

19. $\sum_{n=1}^{\infty} \frac{(x - 2)^n}{n^n}$

20. $\sum_{n=1}^{\infty} \frac{(2x - 1)^n}{5^n \sqrt{n}}$

21. $\sum_{n=1}^{\infty} \frac{n}{b^n} (x - a)^n, \quad b > 0$

22. $\sum_{n=2}^{\infty} \frac{b^n}{\ln n} (x - a)^n, \quad b > 0$

23. $\sum_{n=1}^{\infty} n!(2x - 1)^n$

24. $\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}$

25. $\sum_{n=1}^{\infty} \frac{(5x - 4)^n}{n^3}$

26. $\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$

27. $\sum_{n=1}^{\infty} \frac{x^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n - 1)}$

28. $\sum_{n=1}^{\infty} \frac{n! x^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n - 1)}$

29. If $\sum_{n=0}^{\infty} c_n 4^n$ is convergent, can we conclude that each of the following series is convergent?

(a) $\sum_{n=0}^{\infty} c_n (-2)^n$ (b) $\sum_{n=0}^{\infty} c_n (-4)^n$

30. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series?

(a) $\sum_{n=0}^{\infty} c_n$ (b) $\sum_{n=0}^{\infty} c_n 8^n$

(c) $\sum_{n=0}^{\infty} c_n (-3)^n$ (d) $\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

31. If k is a positive integer, find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$$

with $n = 3$, th

$$\frac{1}{29 \cdot 2^{29}} \approx 6.$$

So we have

$$\int_0^{0.5} \frac{1}{1 + x^7}\, dx \approx \frac{1}{2} - \frac{1}{8 \cdot 2^8} + \cdots$$

11.9 EXERCISES

1. If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 10, what is the radius of convergence of the series $\sum_{n=1}^{\infty} n c_n x^{n-1}$? Why?

2. Suppose you know that the series $\sum_{n=0}^{\infty} b_n x^n$ converges for $|x| < 2$. What can you say about the following series? Why?

$$\sum_{n=0}^{\infty} \frac{b_n}{n+1} x^{n+1}$$

3–10 Find a power series representation for the function and determine the interval of convergence.

3. $f(x) = \dfrac{1}{1+x}$

4. $f(x) = \dfrac{5}{1-4x^2}$

5. $f(x) = \dfrac{2}{3-x}$

6. $f(x) = \dfrac{4}{2x+3}$

7. $f(x) = \dfrac{x^2}{x^4+16}$

8. $f(x) = \dfrac{x}{2x^2+1}$

9. $f(x) = \dfrac{x-1}{x+2}$

10. $f(x) = \dfrac{x+a}{x^2+a^2}, \quad a > 0$

11–12 Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

11. $f(x) = \dfrac{2x-4}{x^2-4x+3}$

12. $f(x) = \dfrac{2x+3}{x^2+3x+2}$

13. (a) Use diff

What is

(b) Use pa

(c) Use p

14. (a) Use

$f(x$

(b) Use

(c) By

as

15–20 Fin

determine

15. $f(x)$

17. $f(x)$

19. $f(x$