doesn't affect the cor
that we were able to show tha

is convergent. Since $\sum_{n=4}^{\infty} \frac{n}{n^3+1}$

$$\sum_{n=1}^{\infty} \frac{n}{n^3+1} = \frac{1}{2} + \frac{2}{9} + \frac{3}{28} + \sum$$

it follows that the entire series $\sum_{n=1}^{\infty} n/(n^3+1)$ is conver
that the series $\sum_{n=N+1}^{\infty} a_n$ converges, then the full series

is also convergent. $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{N} a_n + \sum_{n=N+1}^{\infty} a$

11.2 EXERCISES

1. (a) What is the difference between a sequence and a series?
(b) What is a convergent series? What is a divergent series?

2. Explain what it means to say that $\sum_{n=1}^{\infty} a_n = 5$.

3–4 Calculate the sum of the series $\sum_{n=1}^{\infty} a_n$ whose partial sums are given.

3. $s_n = 2 - 3(0.8)^n$

4. $s_n = \dfrac{n^2-1}{4n^2+1}$

5–8 Calculate the first eight terms of the sequence of partial sums correct to four decimal places. Does it appear that the series is convergent or divergent?

5. $\sum_{n=1}^{\infty} \dfrac{1}{n^4+n^2}$

6. $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt[3]{n}}$

7. $\sum_{n=1}^{\infty} \sin n$

9–14 Find at least 10 part
sequence of terms and the
screen. Does it appear tha
If it is convergent, find th

9. $\sum_{n=1}^{\infty} \dfrac{12}{(-5)^n}$

11. $\sum_{n=1}^{\infty} \dfrac{n}{\sqrt{n^2+4}}$

13. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

14. $\sum_{n=1}^{\infty} \left(\sin\frac{1}{n} - \sin\frac{1}{n+1} \right)$

15. Let $a_n = \dfrac{2n}{3n+1}$.

(a) Determine whether $\{a_n\}$ is convergent.

(b) Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.

16. (a) Explain the difference between

$$\sum_{i=1}^{n} a_i \quad \text{and} \quad \sum_{j=1}^{n} a_j$$

(b) Explain the difference between

$$\sum_{i=1}^{n} a_i \quad \text{and} \quad \sum_{i=1}^{n} a_j$$

17–26 Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

17. $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$

18. $4 + 3 + \frac{9}{4} + \frac{27}{16} + \cdots$

19. $10 - 2 + 0.4 - 0.08 + \cdots$

20. $2 + 0.5 + 0.125 + 0.03125 + \cdots$

21. $\sum_{n=1}^{\infty} 12(0.73)^{n-1}$

22. $\sum_{n=1}^{\infty} \frac{5}{\pi^n}$

23. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

24. $\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n}$

25. $\sum_{n=1}^{\infty} \frac{e^{2n}}{6^{n-1}}$

26. $\sum_{n=1}^{\infty} \frac{6 \cdot 2^{2n-1}}{3^n}$

27–42 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

27. $\frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{12} + \frac{1}{15} + \cdots$

28. $\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \cdots$

29. $\sum_{n=1}^{\infty} \frac{2+n}{1-2n}$

30. $\sum_{k=1}^{\infty} \frac{k^2}{k^2-2k+5}$

31. $\sum_{n=1}^{\infty} 3^{n+1}4^{-n}$

32. $\sum_{n=1}^{\infty} [(-0.2)^n + (0.6)^{n-1}]$

33. $\sum_{n=1}^{\infty} \frac{1}{4+e^{-n}}$

34. $\sum_{n=1}^{\infty} \frac{2^n+4^n}{e^n}$

35. $\sum_{k=1}^{\infty} (\sin 100)^k$

36. $\sum_{n=1}^{\infty} \frac{1}{1+\left(\frac{2}{3}\right)^n}$

37. $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$

38. $\sum_{k=0}^{\infty} \left(\sqrt{2}\right)^{-k}$

39. $\sum_{n=1}^{\infty} \arctan n$

40. $\sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n}\right)$

41. $\sum_{n=1}^{\infty} \left(\frac{1}{e^n} + \frac{1}{n(n+1)}\right)$

42. $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

43–48 Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum (as in Example 8). If it is convergent, find its sum.

43. $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

44. $\sum_{n=1}^{\infty} \ln\frac{n}{n+1}$

45. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

46. $\sum_{n=4}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

47. $\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)}\right)$

48. $\sum_{n=2}^{\infty} \frac{1}{n^3-n}$

49. Let $x = 0.99999\ldots$.

(a) Do you think that $x < 1$ or $x = 1$?

(b) Sum a geometric series to find the value of x.

(c) How many decimal representations does the number 1 have?

(d) Which numbers have more than one decimal representation?

50. A sequence of terms is defined by

$$a_1 = 1 \qquad a_n = (5-n)a_{n-1}$$

Calculate $\sum_{n=1}^{\infty} a_n$.

51–56 Express the number as a ratio of integers.

51. $0.\overline{8} = 0.8888\ldots$

52. $0.\overline{46} = 0.46464646\ldots$

53. $2.\overline{516} = 2.516516516\ldots$

54. $10.1\overline{35} = 10.135353535\ldots$

55. $1.234\overline{567}$

56. $5.\overline{71358}$

57–63 Find the values of x for which the series converges. Find the sum of the series for those values of x.

57. $\sum_{n=1}^{\infty} (-5)^n x^n$

58. $\sum_{n=1}^{\infty} (x+2)^n$

59. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$

60. $\sum_{n=0}^{\infty} (-4)^n (x-5)^n$

61. $\sum_{n=0}^{\infty} \frac{2^n}{x^n}$

62. $\sum_{n=0}^{\infty} \frac{\sin^n x}{3^n}$

63. $\sum_{n=0}^{\infty} e^{nx}$