11.1 EXERCISES

1. (a) What is a sequence?
(b) What does it mean to say that $\lim_{n\to\infty} a_n = 8$?
(c) What does it mean to say that $\lim_{n\to\infty} a_n = \infty$?

2. (a) What is a convergent sequence? Give two examples.
(b) What is a divergent sequence? Give two examples.

3–12 List the first five terms of the sequence.

3. $a_n = \dfrac{2^n}{2n+1}$

4. $a_n = \dfrac{n^2-1}{n^2+1}$

5. $a_n = \dfrac{(-1)^{n-1}}{5^n}$

6. $a_n = \cos\dfrac{n\pi}{2}$

7. $a_n = \dfrac{1}{(n+1)!}$

8. $a_n = \dfrac{(-1)^n n}{n!+1}$

9. $a_1 = 1, \quad a_{n+1} = 5a_n - 3$

10. $a_1 = 6, \quad a_{n+1} = \dfrac{a_n}{n}$

11. $a_1 = 2, \quad a_{n+1} = \dfrac{a_n}{1+a_n}$

12. $a_1 = 2, \quad a_2 = 1, \quad a_{n+1} = a_n - a_{n-1}$

13–18 Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

13. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \dots\right\}$

14. $\left\{4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, \dots\right\}$

15. $\left\{-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \dots\right\}$

16. $\{5, 8, 11, 14, 17, \dots\}$

17. $\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \dots\right\}$

18. $\{1, 0, -1, 0, 1, 0, -1, 0, \dots\}$

19–22 Calculate, to four decimal places, the first ten terms of the sequence and use them to plot the graph of the sequence by hand. Does the sequence appear to have a limit? If so, calculate it. If not, explain why.

19. $a_n = \dfrac{3n}{1+6n}$

20. $a_n = 2 + \dfrac{(-1)^n}{n}$

21. $a_n = 1 + \left(-\frac{1}{2}\right)^n$

22. $a_n = 1 + \dfrac{10^n}{9^n}$

23–56 Determine whether the sequence converges or diverges. If it converges, find the limit.

23. $a_n = \dfrac{3+5n^2}{n+n^2}$

24. $a_n = \dfrac{3+5n^2}{1+n}$

25. $a_n = \dfrac{n^4}{n^3-2n}$

26. $a_n = 2 + (0.86)^n$

27. $a_n = 3^n 7^{-n}$

28. $a_n = \dfrac{3\sqrt{n}}{\sqrt{n}+2}$

29. $a_n = e^{-1/\sqrt{n}}$

30. $a_n = \dfrac{4^n}{1+9^n}$

31. $a_n = \sqrt{\dfrac{1+4n^2}{1+n^2}}$

32. $a_n = \cos\left(\dfrac{n\pi}{n+1}\right)$

33. $a_n = \dfrac{n^2}{\sqrt{n^3+4n}}$

34. $a_n = e^{2n/(n+2)}$

35. $a_n = \dfrac{(-1)^n}{2\sqrt{n}}$

36. $a_n = \dfrac{(-1)^{n+1} n}{n+\sqrt{n}}$

37. $\left\{\dfrac{(2n-1)!}{(2n+1)!}\right\}$

38. $\left\{\dfrac{\ln n}{\ln 2n}\right\}$

39. $\{\sin n\}$

40. $a_n = \dfrac{\tan^{-1} n}{n}$

41. $\{n^2 e^{-n}\}$

42. $a_n = \ln(n+1) - \ln n$

43. $a_n = \dfrac{\cos^2 n}{2^n}$

44. $a_n = \sqrt[n]{2^{1+3n}}$

45. $a_n = n\sin(1/n)$

46. $a_n = 2^{-n}\cos n\pi$

47. $a_n = \left(1 + \dfrac{2}{n}\right)^n$

48. $a_n = \sqrt[n]{n}$

49. $a_n = \ln(2n^2+1) - \ln(n^2+1)$

50. $a_n = \dfrac{(\ln n)^2}{n}$

51. $a_n = \arctan(\ln n)$

52. $a_n = n - \sqrt{n+1}\sqrt{n+3}$

53. $\{0, 1, 0, 0, 1, 0, 0, 0, 1, \dots\}$

54. $\left\{\frac{1}{1}, \frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{6}, \dots\right\}$

56. $a_n = \dfrac{(-3)^n}{n!}$

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8. $a_n = \dfrac{\sin n}{n}$

0. $a_n = \sqrt[n]{3^n + 5^n}$

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68. Find the first 40 terms of the sequence defined by

$$a_{n+1} = \begin{cases} \frac{1}{2}a_n & \text{if } a_n \text{ is an even number} \\ 3a_n + 1 & \text{if } a_n \text{ is an odd number} \end{cases}$$

and $a_1 = 11$. Do the same if $a_1 = 25$. Make a conjecture about this type of sequence.

69. For what values of r is the sequence $\{nr^n\}$ convergent?

70. (a) If $\{a_n\}$ is convergent, show that

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n$$

(b) A sequence $\{a_n\}$ is defined by $a_1 = 1$ and $a_{n+1} = 1/(1 + a_n)$ for $n \geqslant 1$. Assuming that $\{a_n\}$ is convergent, find its limit.

71. Suppose you know that $\{a_n\}$ is a decreasing sequence and all its terms lie between the numbers 5 and 8. Explain why the sequence has a limit. What can you say about the value of the limit?

72–78 Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

72. $a_n = \cos n$

73. $a_n = \dfrac{1}{2n + 3}$

74. $a_n = \dfrac{1 - n}{2 + n}$

75. $a_n = n(-1)^n$

76. $a_n = 2 + \dfrac{(-1)^n}{n}$

77. $a_n = 3 - 2ne^{-n}$

78. $a_n = n^3 - 3n + 3$

79. Find the limit of the sequence

$$\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\right\}$$

80. A sequence $\{a_n\}$ is given by $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$.

(a) By induction or otherwise, show that $\{a_n\}$ is increasing and bounded above by 3. Apply the Monotonic Sequence Theorem to show that $\lim_{n\to\infty} a_n$ exists.

(b) Find $\lim_{n\to\infty} a_n$.

81. Show that the sequence defined by

$$a_1 = 1 \qquad a_{n+1} = 3 - \frac{1}{a_n}$$