## 7.8 EXERCISES

**1.** Explain why each of the following integrals is improper.

(a) $\displaystyle\int_1^2 \frac{x}{x-1}\,dx$

(b) $\displaystyle\int_0^\infty \frac{1}{1+x^3}\,dx$

(c) $\displaystyle\int_{-\infty}^\infty x^2 e^{-x^2}\,dx$

(d) $\displaystyle\int_0^{\pi/4} \cot x\,dx$

**2.** Which of the following integrals are improper? Why?

(a) $\displaystyle\int_0^{\pi/4} \tan x\,dx$

(b) $\displaystyle\int_0^\pi \tan x\,dx$

(c) $\displaystyle\int_{-1}^1 \frac{dx}{x^2-x-2}$

(d) $\displaystyle\int_0^\infty e^{-x^3}\,dx$

**3.** Find the area under the curve $y = 1/x^3$ from $x = 1$ to $x = t$ and evaluate it for $t = 10$, $100$, and $1000$. Then find the total area under this curve for $x \geq 1$.

**4.** (a) Graph the functions $f(x) = 1/x^{1.1}$ and $g(x) = 1/x^{0.9}$ in the viewing rectangles $[0, 10]$ by $[0, 1]$ and $[0, 100]$ by $[0, 1]$.

(b) Find the areas under the graphs of $f$ and $g$ from $x = 1$ to $x = t$ and evaluate for $t = 10$, $100$, $10^4$, $10^6$, $10^{10}$, and $10^{20}$.

(c) Find the total area under each curve for $x \geq 1$, if it exists.

**5–40** Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

**5.** $\displaystyle\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx$

**6.** $\displaystyle\int_0^\infty \frac{1}{\sqrt[4]{1+x}}\,dx$

**7.** $\displaystyle\int_{-\infty}^0 \frac{1}{3-4x}\,dx$

**8.** $\displaystyle\int_1^\infty \frac{1}{(2x+1)^3}\,dx$

**9.** $\displaystyle\int_2^\infty$

**11.** $\displaystyle\int_0^\infty$

**13.** $\displaystyle\int_-^\infty$

**15.** $\displaystyle\int_0^\infty$

**17.** $\displaystyle\int_1^\infty$

**19.** $\displaystyle\int$

**21.** $\displaystyle\int$

**23.** $\displaystyle\int$

**25.** $\displaystyle\int$

**27.**

**29.**

**31.**

2 illustrates the divergence of the integral in Example 10. It appears that the not approaching any fixed number.

s improper.

dx

x

? Why?

$= 1$ to $x = t$

n find the total

$) = 1/x^{0.9}$ in

d $[0, 100]$

from $x = 1$

$10^6$, $10^{10}$,

1, if it exists.

nt or divergent.

dx

$\frac{}{3} dx$

**9.** $\int_{2}^{\infty} e^{-5p} \, dp$

**11.** $\int_{0}^{\infty} \frac{x^2}{\sqrt{1 + x^3}} \, dx$

**13.** $\int_{-\infty}^{\infty} xe^{-x^2} \, dx$

**15.** $\int_{0}^{\infty} \sin^2\alpha \, d\alpha$

**17.** $\int_{1}^{\infty} \frac{1}{x^2 + x} \, dx$

**19.** $\int_{-\infty}^{0} ze^{2z} \, dz$

**21.** $\int_{1}^{\infty} \frac{\ln x}{x} \, dx$

**23.** $\int_{-\infty}^{0} \frac{z}{z^4 + 4} \, dz$

**25.** $\int_{0}^{\infty} e^{-\sqrt{y}} \, dy$

**27.** $\int_{0}^{1} \frac{1}{x} \, dx$

**29.** $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x + 2}}$

**31.** $\int_{-2}^{3} \frac{1}{x^4} \, dx$

**10.** $\int_{-\infty}^{0} 2^r \, dr$

**12.** $\int_{-\infty}^{\infty} (y^3 - 3y^2) \, dy$

**14.** $\int_{1}^{\infty} \frac{e^{-1/x}}{x^2} \, dx$

**16.** $\int_{0}^{\infty} \sin\theta \, e^{\cos\theta} \, d\theta$

**18.** $\int_{2}^{\infty} \frac{dv}{v^2 + 2v - 3}$

**20.** $\int_{2}^{\infty} ye^{-3y} \, dy$

**22.** $\int_{1}^{\infty} \frac{\ln x}{x^2} \, dx$

**24.** $\int_{e}^{\infty} \frac{1}{x(\ln x)^2} \, dx$

**26.** $\int_{1}^{\infty} \frac{dx}{\sqrt{x} + x\sqrt{x}}$

**28.** $\int_{0}^{5} \frac{1}{\sqrt[3]{5 - x}} \, dx$

**30.** $\int_{-1}^{2} \frac{x}{(x + 1)^2} \, dx$

**32.** $\int_{0}^{1} \frac{dx}{\sqrt{1 - x^2}}$

**33.** $\displaystyle\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx$

**34.** $\displaystyle\int_0^5 \frac{w}{w-2}\,dw$

**35.** $\displaystyle\int_0^{\pi/2} \tan^2\theta\,d\theta$

**36.** $\displaystyle\int_0^4 \frac{dx}{x^2-x-2}$

**37.** $\displaystyle\int_0^1 r\ln r\,dr$

**38.** $\displaystyle\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{\sin\theta}}\,d\theta$

**39.** $\displaystyle\int_{-1}^0 \frac{e^{1/x}}{x^3}\,dx$

**40.** $\displaystyle\int_0^1 \frac{e^{1/x}}{x^3}\,dx$

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**41–46** Sketch the region and find its area (if the area is finit

**41.** $S = \{(x,y) \mid x \geq 1,\ 0 \leq y \leq e^{-x}\}$

**42.** $S = \{(x,y) \mid x \leq 0,\ 0 \leq y \leq e^x\}$

**43.** $S = \{(x,y) \mid x \geq 1,\ 0 \leq y \leq 1/(x^3+x)\}$

**44.** $S = \{(x,y) \mid x \geq 0,\ 0 \leq y \leq xe^{-x}\}$

**45.** $S = \{(x,y) \mid 0 \leq x < \pi/2,\ 0 \leq y \leq \sec^2 x\}$

**46.** $S = \{(x,y) \mid -2 < x \leq 0,\ 0 \leq y \leq 1/\sqrt{x+2}\}$