Review Packet #3 Math 12

1. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\dfrac{(2x+3)^{n+1}}{n+1}}{\dfrac{(2x+3)^n}{n}}\right| = \lim_{n\to\infty}\left|\dfrac{(2x+3)^{n+1}}{(2x+3)^n}\right|\cdot\dfrac{n}{n+1} = |2x+3|$ if <1 get convergence by Ratio Test

Check: $|2x+3|<1 \rightarrow -1<2x+3<1$

$-4<2x<-2$

$-2<x<-1$

Endpoints:

- $x=-2$ Series Becomes $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ which is the convergent Alternating Harmonic Series

Convergent by Alternating Series Test

$$\begin{cases} ① \ b_n = \frac{1}{n} > 0 \\ ② \ \lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{1}{n} = 0 \\ ③ \ b_{n+1} < b_n \quad \frac{1}{n+1} < \frac{1}{n} \ ✓ \end{cases}$$

- $x=-1$ Series becomes $\sum_{n=1}\frac{1}{n}$ which is divergent Harmonic Series with $p=1$

Finally, the Interval of Convergence $\boxed{I=[-2,-1)}$ with Radius of Convergence $\boxed{R=1/2}$

centered at $-3/2$

2. Ratio Test

$\lim_{n\to\infty}\left|\dfrac{\dfrac{(-3)^{n+1}x^{n+1}}{(n+1)^2 4^{n+1}}}{\dfrac{(-3)^n x^n}{n^2 4^n}}\right| = \lim_{n\to\infty}\left|\dfrac{(-3)^{n+1}}{(-3)^n}\cdot\dfrac{x^{n+1}}{x^n}\right|\cdot\dfrac{4^n}{4^{n+1}}\cdot\dfrac{n^2}{(n+1)^2} = \dfrac{3}{4}|x|$ by R.T. convergent if <1.

Check: $\frac{3}{4}|x|<1 \Rightarrow |x|<\frac{4}{3} \Rightarrow -\frac{4}{3}<x<\frac{4}{3}$

Endpoints: • $x=4/3$ Series Becomes $\sum_{n=1}^{\infty}\frac{(-3)^n(4/3)^n}{n^2 4^n} = \sum_{n=1}^{\infty}\frac{(-1)^n 3^n 4^n}{n^2 4^n 3^n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ which is convergent either By using Alternating Series Test, but quicker to use Absolute Convergence Test since we know $\sum_{n=1}\frac{1}{n^2}$ convergent p-series with $p=2>1$

- $x=-4/3$

Series Becomes $\sum_{n=1}^{\infty}\frac{(-3)^n(-\frac{4}{3})^n}{n^2 4^n} = \sum_{n=1}^{\infty}\frac{(-1)^n 3^n(-1)^n 4^n}{n^2 4^n 3^n} = \sum_{n=1}^{\infty}\frac{1}{n^2}$ which is convergent p-series $p=2>1$

Finally, the Interval of Convergence $\boxed{I=[-4/3,\ 4/3]}$ with $\boxed{R=4/3}$

3. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\frac{10^{n+1}(x+3)^{n+1}}{(n+2)^3(n+1)!}}{\frac{10^n(x+3)^n}{(n+1)^3 n!}}\right| = \lim_{n\to\infty}\dfrac{10^{n+1}}{10^n}\left|\dfrac{(x+3)^{n+1}}{(x+3)^n}\right|\left[\dfrac{\cancel{n+1}}{\cancel{n+2}}\right]^3\dfrac{\cancel{n!}}{\frac{(n+1)!}{(n+1)\cancel{n!}}} = \lim_{n\to\infty}\dfrac{10|x+3|}{n+1} = 0 < 1$

Converges by RT for all

Finally, $\boxed{I = (-\infty, \infty)}$, $\boxed{R = \infty}$

4. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\frac{(n+1)2^{n+1}(x+1)^{n+1}}{n+6}}{\frac{n\,2^n(x+1)^n}{n+5}}\right| = \lim_{n\to\infty}\dfrac{\cancel{n+1}}{\cancel{n}}\,\dfrac{2^{n+1}}{2^n}\left|\dfrac{(x+1)^{n+1}}{(x+1)^n}\right|\dfrac{\cancel{n+5}}{\cancel{n+6}} = 2|x+1| < 1$ get Convergence by Ratio Test

$\hookrightarrow |x+1| < \frac{1}{2}$

$-\frac{1}{2} < x+1 < \frac{1}{2}$

$-\frac{3}{2} < x < -\frac{1}{2}$

Endpoints: • $x = -\frac{3}{2}$ Original Series becomes

$\sum_{n=0}^{\infty}\dfrac{n2^n}{n+5}\left(\dfrac{-1}{2}\right)^n = \sum_{n=0}^{\infty}\dfrac{(-1)^n n\cancel{2^n}}{n+5\ \cancel{2^n}} = \sum\dfrac{(-1)^n n}{n+5}$

Diverges since terms don't go to zero, $\lim_{n\to\infty}\dfrac{(-1)^n n}{n+5}$ D.N.E. $\neq 0$. by n^{th} Term Divergence Test

• $x = -\frac{1}{2}$ Original Series becomes

$\sum_{n=0}^{\infty}\dfrac{n\,2^n}{n+5}\left(\dfrac{1}{2}\right)^n = \sum_{n=0}^{\infty}\dfrac{n}{n+5}\dfrac{\cancel{2^n}}{\cancel{2^n}} = \sum_{n=0}^{\infty}\dfrac{n}{n+5}$

Diverges since terms don't go to zero, $\lim_{n\to\infty}\dfrac{n}{n+5} = 1 \neq 0$

by n^{th} Term Divergence Test

Finally, $\boxed{I = (-\frac{3}{2}, -\frac{1}{2})}$ $\boxed{R = \frac{1}{2}}$

$-\frac{3}{2}$ $\quad -1 \quad$ $-\frac{1}{2}$

5. Ratio test $\lim_{n\to\infty}\left|\dfrac{\frac{(n+3)!\,(x-5)^{n+1}}{10^{n+1}}}{\frac{(n+2)!\,(x-5)^n}{10^n}}\right| = \lim_{n\to\infty}\dfrac{\overset{(n+3)\cancel{(n+2)!}}{(n+3)!}}{\cancel{(n+2)!}}\left|\dfrac{(x-5)^{n+1}}{(x-5)^n}\right|\dfrac{10^n}{10^{n+1}} = \lim_{n\to\infty}\dfrac{n+3}{10}|x-5| = \infty > 1$ Diverges by R.T. unless $x = 5$.

Interval of Convergence $\boxed{\{x=5\}}$ $\boxed{R=0}$

6. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\frac{\sqrt{n+1}\,(2x-1)^{n+1}}{4^{n+1}}}{\frac{\sqrt{n}\,(2x-1)^n}{4^n}}\right| = \lim_{n\to\infty}\dfrac{\cancel{\sqrt{n+1}}}{\cancel{\sqrt{n}}}\left|\dfrac{(2x-1)^{n+1}}{(2x-1)^n}\right|\dfrac{4^n}{4^{n+1}} = \dfrac{|2x-1|}{4}$ if < 1 get convergence by Ratio Test.

$|2x-1| < 4$

$-4 < 2x-1 < 4$

$-3 < 2x < 5$

$-\frac{3}{2} < x < \frac{5}{2}$

Endpoints:

• $x = \frac{5}{2}$ Original Series becomes

$\sum_{n=0}^{\infty}\dfrac{\sqrt{n}\,\cancel{4^n}}{\cancel{4^n}} = \sum_{n=0}^{\infty}\sqrt{n}$ Diverges n^{th} Term Divergence Test

$\lim_{n\to\infty}\sqrt{n} = \infty \neq 0$

$-\frac{3}{2}$ $\quad \frac{1}{2} \quad$ $\frac{5}{2}$

• $x = -\frac{3}{2}$ Original Series becomes

$\boxed{\text{Interval} = (-\frac{3}{2}, \frac{5}{2})}$ $\boxed{R = 2}$

$\sum_{n=0}^{\infty}\dfrac{\sqrt{n}(-4)^n}{4^n} = \sum_{n=0}^{\infty}(-1)^n\sqrt{n}$ Diverges by n^{th} Term Divergence Test $\lim_{n\to\infty}(-1)^n\sqrt{n}$ D.N.E $\neq 0$

7. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\frac{(-1)^{n+1}x^{2(n+1)+1}}{(2(n+1)+1)!}}{\frac{(-1)^n x^{2n+1}}{(2n+1)!}}\right| = \lim_{n\to\infty}\left|\dfrac{x^{2n+3}}{x^{2n+1}}\right|\dfrac{\cancel{(2n+1)!}}{\underbrace{(2n+3)!}_{(2n+3)(2n+2)\cancel{(2n+1)!}}} = \lim_{n\to\infty}\dfrac{|x|^2}{(2n+3)(2n+2)} = 0 < 1$ by R.T. converges for all x.

$\boxed{\text{Interval} = (-\infty, \infty)}$ $\boxed{R = \infty}$

Note: this is the Maclaurin Series for sin x and this work shows that Power series had convergence for all x.

8. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\frac{(-1)^{n+1}x^{n+1}}{(n+1)^{n+1}}}{\frac{(-1)^n x^n}{n^n}}\right| = \lim_{n\to\infty}\left|\dfrac{x^{n+1}}{x^n}\right|\dfrac{n^n}{\underbrace{(n+1)^{n+1}}_{(n+1)(n+1)^n}} = \lim_{n\to\infty}\dfrac{|x|}{n+1}\underbrace{\left(\dfrac{n}{n+1}\right)^n}_{\frac{1}{e}} = 0 < 1$ by R.T. converges for all x.

$\boxed{\text{Interval} = (-\infty, \infty)}$ $\boxed{R = \infty}$

9. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\frac{\ln(n+1)\,x^{n+1}}{(n+1)^2}}{\frac{\ln n\; x^n}{n^2}}\right| = \lim_{n\to\infty}\underbrace{\dfrac{\ln(n+1)}{\ln n}}_{1}\left|\dfrac{x^{n+1}}{x^n}\right|\underbrace{\dfrac{n^2}{(n+1)^2}}_{1} = |x|$ if <1 get convergence by R.T.

$|x| < 1$

$-1 < x < 1$

Note: $\lim_{n\to\infty}\dfrac{\ln(n+1)}{\ln n}\ \dfrac{\infty}{\infty} \overset{L'H}{=} \lim_{n\to\infty}\dfrac{\frac{1}{n+1}}{\frac{1}{n}} = \lim_{n\to\infty}\dfrac{n}{n+1} = 1$

Endpoints:

- $x = 1$ Original Series becomes $\sum_{n=2}^{\infty}\dfrac{\ln n}{n^2}$

  We bound the terms $\dfrac{\ln n}{n^2} < \dfrac{\sqrt{n}}{n^2} = \dfrac{1}{n^{3/2}}$ and since $\sum_{n=2}^{\infty}\dfrac{1}{n^{3/2}}$ is a convergent p-series $p = \frac{3}{2} > 1$

  then $\sum_{n=2}\dfrac{\ln n}{n^2}$ also converges by Comparison Test (CT)

- $x = -1$ Original Series becomes $\sum_{n=2}^{\infty}\dfrac{(-1)^n \ln n}{n^2}$ which converges by Absolute Convergence Test since $\sum\frac{\ln n}{n^2}$ Conv. as above.

$\boxed{I = [-1, 1]}$ $\boxed{R = 1}$

10. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\frac{(2n+2)!\,x^{n+1}}{(3n+3)!}}{\frac{(2n)!\,x^n}{(3n)!}}\right| = \lim_{n\to\infty}\dfrac{\overbrace{(2n+2)!}^{(2n+2)(2n+1)\cancel{(2n)!}}\,\cancel{(3n)!}}{\cancel{(2n)!}\,\underbrace{(3n+3)!}_{(3n+3)(3n+2)(3n+1)\cancel{(3n)!}}}\left|\dfrac{x^{n+1}}{x^n}\right| = \lim_{n\to\infty}\underbrace{\dfrac{(2n+2)(2n+1)}{(3n+3)(3n+2)(3n+1)}}_{0}|x| = 0 < 1$ by RT converges for all x

$\boxed{\text{Interval} = (-\infty, \infty)}$ $\boxed{R = \infty}$

11. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\dfrac{(-1)^{n+1}(n+1)!(x-1)^{n+1}}{(n+1)^3}}{\dfrac{(-1)^n n!(x-1)^n}{n^3}}\right| = \lim_{n\to\infty}\dfrac{\overbrace{(n+1)!}^{(n+1)\,n!}}{\cancel{n!}}\;\dfrac{n^3}{(n+1)^3}\left|\dfrac{(x-1)^{n+1}}{(x-1)^n}\right| = \lim_{n\to\infty}(n+1)|x-1| = \infty > 1$ unless $x=1$

Diverges by R.T.

$\boxed{\text{Interval} = \{x=1\}}$ or $\{1\}$ $\boxed{R=0}$

12. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\dfrac{x^{n+1}}{\sqrt{n+1}}}{\dfrac{x^n}{\sqrt{n}}}\right| = \lim_{n\to\infty}\left|\dfrac{x^{n+1}}{x^n}\right|\dfrac{\sqrt{n}}{\sqrt{n+1}} = |x|$ if <1 get convergence by R.T.

$\left(\sqrt{\dfrac{n}{n+1}}\right)$ $-1<x<1$

Endpoints • $x=1$ Original Series becomes $\sum_{n=1}^{\infty}\dfrac{1}{\sqrt{n}}$ Divergent p-series $p=\frac{1}{2}<1$

• $x=-1$ Original Series becomes $\sum_{n=1}^{\infty}\dfrac{(-1)^n}{\sqrt{n}}$ Converges by AST

① $b_n=\dfrac{1}{\sqrt{n}}>0$

② $\lim_{n\to\infty} b_n=\lim_{n\to\infty}\dfrac{1}{\sqrt{n}}=0$

③ $b_{n+1}<b_n$ $\dfrac{1}{\sqrt{n+1}}<\dfrac{1}{\sqrt{n}}$ since $\sqrt{x}$ increasing function.

$\boxed{\text{Interval } [-1,1)}$ $\boxed{R=1}$

13. Ratio Test $\lim_{n\to\infty}\left|\dfrac{(n+1)x^{n+1}}{n\,x^n}\right| = |x|$ if <1 get convergence

$-1<x<1$

Endpoints • $x=1$ Original Series becomes $\sum_{n=1}^{\infty} n$ Divergent by n^{th} term Div. Test $\lim_{n\to\infty} n=\infty\neq 0$

• $x=-1$ Original Series becomes $\sum_{n=1}^{\infty}(-1)^n n$ Divergent by n^{th} term Div. Test $\lim_{n\to\infty}(-1)^n n$ D.N.E. $\neq 0$

$\boxed{\text{Interval } (-1,1)}$ $\boxed{R=1}$

14. Ratio Test $\lim_{n\to\infty}\left|\dfrac{\dfrac{(n+1)!\,x^{n+1}}{(n+1)^{n+1}}}{\dfrac{n!\,x^n}{n^n}}\right| = \lim_{n\to\infty}\dfrac{\overbrace{(n+1)!}^{(n+1)\,n!}}{\cancel{n!}}\left|\dfrac{x^{n+1}}{x^n}\right|\dfrac{n^n}{\underbrace{(n+1)^{n+1}}_{(n+1)(n+1)^n}} = \lim_{n\to\infty}|x|\left(\dfrac{n}{n+1}\right)^n = \dfrac{|x|}{e}$ if <1 get convergence by R.T.

$|x|<e$

$-e<x<e$

Challenge Part Here

Endpoints • $x=e$ Original Series becomes $\sum_{n=1}^{\infty}\dfrac{n!\,e^n}{n^n}$ which turns out to be Divergent by n^{th} term Divergence Test since $\lim_{n\to\infty}\dfrac{n!\,e^n}{n^n}\neq 0$.

Why? First note: $e^x = 1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\cdots+\dfrac{x^n}{n!}+\cdots$

$\Rightarrow e^n = 1+n+\dfrac{n^2}{2!}+\dfrac{n^3}{3!}+\cdots+\dfrac{n^n}{n!}+\cdots$ From this we know $e^n \geq \dfrac{n^n}{n!} \Rightarrow \dfrac{n!}{n^n}\cdot e^n \geq 1$

only one term in sum

$\Rightarrow \lim_{n\to\infty}\dfrac{n!\,e^n}{n^n}\neq 0$ ✓

• $x=-e$ Original Series becomes $\sum_{n=1}^{\infty}\dfrac{(-1)^n n!\,e^n}{n^n}$

From last step, $\lim_{n\to\infty}\dfrac{n!\,e^n}{n^n}\neq 0 \Rightarrow \lim_{n\to\infty}\dfrac{(-1)^n n!\,e^n}{n^n}$ D.N.E. $\Rightarrow$ this Series also Diverges by n^{th} Term Div. Test

Finally. $\boxed{\text{Interval} = (-e,e)}$ $\boxed{R=e}$

15. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$$\cos 1 = 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \cdots$$

$$= 1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720} + \cdots$$

$$\approx 1 - \frac{1}{2} + \frac{1}{24} = \frac{24}{24} - \frac{12}{24} + \frac{1}{24} = \boxed{\frac{13}{24}}$$ ← Estimate using AS ET, error is at most $\frac{1}{720} < \frac{1}{100}$ as desired

16. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$$e^{-1/3} = 1 - 1/3 + \frac{(1/3)^2}{2!} + \frac{(-1/3)^3}{3!} - \cdots$$

$27 \times 6 = 162$

$$= 1 - 1/3 + \frac{1}{18} - \frac{1}{162} + \cdots$$

$$\approx 1 - \frac{1}{3} + \frac{1}{18} = \frac{18}{18} - \frac{6}{18} + \frac{1}{18} = \boxed{\frac{13}{18}}$$ ← Estimate using ASET, error is at most $\frac{1}{162} < \frac{1}{100}$ as desired

17. $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

$$\arctan 1 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

$$\approx 1 - \frac{1}{3} + \frac{1}{5} = \frac{15}{15} - \frac{5}{15} + \frac{3}{15} = \boxed{\frac{13}{15}}$$ ← Estimate using ASET, error is at most $\frac{1}{7} < \frac{1}{5} = .20$ as desired

18. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

$$e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \cdots$$

$$= 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} + \cdots$$

$$\approx \cancel{1-1} + \frac{1}{2} - \frac{1}{6} = \frac{1}{2} - \frac{1}{6} = \frac{3}{6} - \frac{1}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$$ ← Estimate using ASET, error is at most $\frac{1}{24} < \frac{1}{10}$ as desired

19. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\sin 1 = 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \cdots$

$= 1 - \frac{1}{6} + \frac{1}{120} - \cdots$

$\approx 1 - \frac{1}{6} = \boxed{\frac{5}{6}}$ ← Estimate using ASET, error at most $\frac{1}{120} < \frac{1}{100}$ as desired

20. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

$\frac{1}{\sqrt{e}} = e^{-1/2} = 1 - \frac{1}{2} + \frac{(-1/2)^2}{2!} + \frac{(-1/2)^3}{3!} + \frac{(-1/2)^4}{4!} + \cdots$

$= 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} + \frac{1}{384} - \cdots$

$\approx 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} = \frac{48}{48} - \frac{24}{48} + \frac{6}{48} - \frac{1}{48} = \boxed{\frac{29}{48}}$ ← Estimate using ASET, error is at most $\frac{1}{384} < \frac{1}{100}$ as desired.

21. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\sin\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^3}{3!} + \frac{\left(\frac{1}{2}\right)^5}{5!} - \frac{\left(\frac{1}{2}\right)^7}{7!} + \cdots$

$= \frac{1}{2} - \frac{1}{48} + \frac{1}{3{,}840} - \cdots$

$\approx \frac{1}{2} - \frac{1}{48} = \frac{24}{48} - \frac{1}{48} = \boxed{\frac{23}{48}}$ ← Estimate using ASET, error is at most $\frac{1}{3840} < \frac{1}{100}$ as desired

22. $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

$$\arctan\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^3}{3} + \frac{\left(\frac{1}{2}\right)^5}{5} - \frac{\left(\frac{1}{2}\right)^7}{7} + \dots$$

$$= \frac{1}{2} - \frac{1}{24} + \frac{1}{160} - \dots$$

$$\approx \frac{1}{2} - \frac{1}{24} = \frac{12}{24} - \frac{1}{24} = \boxed{\frac{11}{24}}$$

← Estimate using ASET, error at most $\frac{1}{160} < \frac{1}{100}$ as desired

23. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \dots$

$$\ln 2 = \ln(1+1) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$$

$$\approx 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} = \frac{60}{60} - \frac{30}{60} + \frac{20}{60} - \frac{15}{60} + \frac{12}{60}$$

$$= \boxed{\frac{47}{60}}$$

Estimate using ASET, error at most $\frac{1}{6} < \frac{1}{5}$ as desired

24. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

$$\cos\left(\frac{1}{2}\right) = 1 - \frac{\left(\frac{1}{2}\right)^2}{2!} + \frac{\left(\frac{1}{2}\right)^4}{4!} - \frac{\left(\frac{1}{2}\right)^6}{6!} + \dots$$

$$= 1 - \frac{1}{8} + \frac{1}{384} - \dots$$

$$\approx 1 - \frac{1}{8} = \boxed{\frac{7}{8}}$$

← Estimate using ASET, error at most $\frac{1}{384} < \frac{1}{100}$ as desired

25. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

$$\ln\left(\frac{3}{2}\right) = \ln\left(1 + \frac{1}{2}\right) = \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^2}{2} + \frac{\left(\frac{1}{2}\right)^3}{3} - \frac{\left(\frac{1}{2}\right)^4}{4} + \dots$$

$$= \frac{1}{2} - \frac{1}{8} + \frac{1}{24} - \dots$$

$$\approx \frac{1}{2} - \frac{1}{8} = \frac{4}{8} - \frac{1}{8} = \boxed{\frac{3}{8}}$$

← Estimate using ASET, error at most $\frac{1}{24} < \frac{1}{10}$ as desired.

26. $f(x) = x^2 \underbrace{e^{-3x^4}}_{R=\infty} = x^2 \sum_{n=0}^{\infty} \frac{(-3x^4)^n}{n!} = x^2 \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{4n}}{n!} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{4n+2}}{n!}}$ $\boxed{R=\infty}$

27. $f(x) = \frac{1-e^{-x}}{x} = \frac{1 - \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}}{x} = \frac{1 - \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}}{x}$ what will cancel?

$= \frac{\cancel{1} - (\cancel{1} - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots)}{x} = \frac{x - \frac{x^2}{2!} + \frac{x^3}{3!} - \dots}{x}$

n=0 cancel

$= 1 - \frac{x}{2!} + \frac{x^2}{3!} - \dots = \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n-1}}{n!}}$ $\boxed{R=\infty}$

OR $\frac{1 - \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}}{x} = \frac{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n!}}{x} = \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n-1}}{n!}}$

28. $x^4 \ln(1+x^3) = x^4 \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{n+1}}{n+1} = x^4 \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+3}}{n+1}$

Need $|x^3| < 1$
$\Rightarrow |x| < 1$
$\Rightarrow R = 1$ o.k.

$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+7}}{n+1}}$ $\boxed{R=1}$

Two Main Options

29. $\cosh x = \frac{e^x + e^{-x}}{2} = \frac{1}{2}\left[\sum_{n=0}^{\infty} \frac{x^n}{n!} + \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}\right]$

$= \frac{1}{2}\left[\sum_{n=0}^{\infty} \frac{x^n}{n!} + \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}\right]$

$= \frac{1}{2}\left[1 + \cancel{x} + \frac{x^2}{2!} + \cancel{\frac{x^3}{3!}} + \dots + 1 - \cancel{x} + \frac{x^2}{2!} - \cancel{\frac{x^3}{3!}} + \dots\right]$ all odd powers cancel

$= \cancel{\frac{1}{2}}\left[\cancel{2} + \cancel{2}\left(\frac{x^2}{2!}\right) + \cancel{2}\left(\frac{x^4}{4!}\right) + \dots\right]$ 2's cancel

NOT alternating like cosine does

$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots = \boxed{\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}}$ $\boxed{R = \infty}$ b/c each exponential has $R = \infty$

OR use Ratio Test...

OR Use Chart Method for MacLaurin Series

$f(x) = \cosh x \qquad f(0) = \cosh 0 = 1$

$f'(x) = \sinh x \qquad f'(0) = \sinh 0 = 0$

$f''(x) = \cosh x \qquad f''(0) = \cosh 0 = 1$

$f'''(x) = \sinh x \qquad f'''(0) = \sinh 0 = 0$

$\vdots \qquad \vdots$

MacLaurin Series

$\underbrace{f(0)}_{1} + \underbrace{f'(0)}_{0}x + \frac{\overbrace{f''(0)}^{1}x^2}{2!} + \frac{\overbrace{f'''(0)}^{0}x^3}{3!} + \frac{\overbrace{f^{(4)}(0)}^{1}x^4}{4!} + \dots$

$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots = \boxed{\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}}$

Again, multiple options

From # 29.

30. $f(x) = \sinh x = \frac{d}{dx}\cosh x = \frac{d}{dx}\left[\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}\right] = \frac{d}{dx}\left[1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots\right]$

$$= 0 + \frac{1}{\cancel{2}!}(\cancel{2}x) + \frac{1}{\underbrace{4!}_{4\cdot 3!}}(\cancel{4}x^3) + \frac{1}{\underbrace{6!}_{6\cdot 5!}}(\cancel{6}x^5) + \dots$$

$$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots = \boxed{\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}} \quad \boxed{R=\infty} \text{ STILL}$$

OR $\sinh x = \frac{e^x - e^{-x}}{2} = \frac{1}{2}\left[\sum_{n=0}^{\infty} \frac{x^n}{n!} - \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}\right] = \frac{1}{2}\left[\sum_{n=0}^{\infty} \frac{x^n}{n!} - \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}\right]$

$$= \frac{1}{2}\left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots - \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots\right)\right]$$

All even powers cancel

$$= \frac{1}{\cancel{2}}\left[\cancel{2}x + \frac{\cancel{2}x^3}{3!} + \frac{\cancel{2}x^5}{5!} + \dots\right] \quad \text{2's Cancel}$$

$$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots = \boxed{\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}}$$

R=∞ b/c both exponentials have R=∞

OR Use Chart Method

$f(x) = \sinh x \qquad f(0) = \sinh 0 = 0$

$f'(x) = \cosh x \qquad f'(0) = \cosh 0 = 1$

$f''(x) = \sinh x \qquad f''(0) = \sinh 0 = 0$

$f'''(x) = \cosh x \qquad f'''(0) = \cosh 0 = 1$

$\vdots \qquad\qquad \vdots$

Maclaurin Series

$$\cancel{f(0)}^{0} + \cancel{f'(0)}^{1} x + \frac{\cancel{f''(0)}^{0} x^2}{2!} + \frac{\cancel{f'''(0)}^{1} x^3}{3!} + \frac{\cancel{f^{(4)}(0)}^{0} x^4}{4!} + \dots$$

$$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

Match Again!

31. $f(x) = \dfrac{x^6}{1+7x} = x^6\left[\dfrac{1}{1+7x}\right] = x^6\left[\dfrac{1}{1-(-7x)}\right] = x^6 \underbrace{\sum_{n=0}^{\infty}(-7x)^n}$

Geometric, Needs
$|-7x| = |7x| < 1$
$\Rightarrow |x| < 1/7$ $\quad \boxed{R = \frac{1}{7}}$

$= x^6 \sum_{n=0}^{\infty}(-1)^n 7^n x^n = \boxed{\sum_{n=0}^{\infty}(-1)^n 7^n x^{n+6}}$

32. $f(x) = x\arctan(2x) = x\sum_{n=0}^{\infty}\dfrac{(-1)^n (2x)^{2n+1}}{2n+1}$ Need $|2x|<1 \Rightarrow |x|<\frac{1}{2}$ $\boxed{R=\frac{1}{2}}$

$= x\sum_{n=0}^{\infty}\dfrac{(-1)^n 2^{2n+1} x^{2n+1}}{2n+1} = \boxed{\sum_{n=0}^{\infty}\dfrac{(-1)^n 2^{2n+1} x^{2n+2}}{2n+1}}$

33. $\int_0^1 x^2\cos(x^3)\,dx = \int_0^1 x^2\sum_{n=0}^{\infty}\dfrac{(-1)^n(x^3)^{2n}}{(2n)!}\,dx = \int_0^1 x^2\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{6n}}{(2n)!}\,dx$

$= \int_0^1 \sum_{n=0}^{\infty}\dfrac{(-1)^n x^{6n+2}}{(2n)!}\,dx = \sum_{n=0}^{\infty}\dfrac{(-1)^n x^{6n+3}}{(2n)!\,(6n+3)}\Bigg|_0^1$

$= \dfrac{x^3}{1\cdot 3} - \dfrac{x^9}{2!\cdot 9} + \dfrac{x^{15}}{4!\cdot(15)} - \cdots\Bigg|_0^1 = \dfrac{1}{3} - \dfrac{1}{18} + \dfrac{1}{360} - \cdots$

$\approx \dfrac{1}{3} - \dfrac{1}{18} = \dfrac{6}{18} - \dfrac{1}{18} = \boxed{\dfrac{5}{18}}$ ← Estimate

Using ASET we can estimate the full sum using only the first two terms with error at most the absolute value of the first neglected term $\frac{1}{360} < \frac{1}{50}$ as desired.

34. $\int_0^{1/2} x\arctan x\,dx = \int_0^{1/2} x\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}\,dx = \int_0^{1/2}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+2}}{2n+1}\,dx$

$= \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+3}}{(2n+1)(2n+3)}\Bigg|_0^{1/2} = \frac{x^3}{1\cdot 3} - \frac{x^5}{3\cdot 5} + \frac{x^7}{5\cdot 7} - \ldots\Bigg|_0^{1/2}$

$= \frac{\left(\frac{1}{2}\right)^3}{3} - \frac{\left(\frac{1}{2}\right)^5}{15} + \frac{\left(\frac{1}{2}\right)^7}{35} - \ldots - (0-0+0-\ldots)$

$= \frac{1}{24} - \frac{1}{480} + \ldots \approx \boxed{\frac{1}{24}}$ ← Estimate.

Using ASET we can estimate full sum as $\frac{1}{24}$ with error at most $\frac{1}{480} < \frac{1}{100} = 0.01$ as desired.

35. $\int_0^1 \sin(x^2)\,dx = \int_0^1 \sum_{n=0}^{\infty}\frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!}\,dx = \int_0^1 \sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+2}}{(2n+1)!}\,dx$

$= \sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+3}}{(2n+1)!\,(4n+3)}\Bigg|_0^1 = \frac{x^3}{1!\cdot 3} - \frac{x^7}{3!\cdot 7} + \frac{x^{11}}{5!\cdot(11)} - \ldots\Bigg|_0^1$

$= \frac{1}{3} - \frac{1}{42} + \ldots - (0-0+\ldots) \approx \boxed{\frac{1}{3}}$ ← Estimate.

Using ASET, we can estimate Full Sum as $1/3$ with error at most $\frac{1}{42} < \frac{1}{10} = 0.1$ as desired.

36. $\int_0^{1/2} e^{-x^3}\,dx = \int_0^{1/2}\sum_{n=0}^{\infty}\frac{(-x^3)^n}{n!}\,dx = \int_0^{1/2}\sum_{n=0}^{\infty}\frac{(-1)^n x^{3n}}{n!}\,dx = \sum_{n=0}^{\infty}\frac{(-1)^n x^{3n+1}}{n!\,(3n+1)}\Bigg|_0^{1/2}$

$= x - \frac{x^4}{1!\cdot 4} + \frac{x^7}{2!\cdot 7} - \frac{x^{10}}{3!\cdot(10)} + \ldots\Bigg|_0^{1/2} = \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^4}{4} + \frac{\left(\frac{1}{2}\right)^7}{14} - \ldots - (0-0+0-\ldots)$

$= \frac{1}{2} - \frac{1}{64} + \frac{1}{1792} - \ldots \approx 1/2 - 1/64 = \frac{32}{64} - \frac{1}{64} = \boxed{\frac{31}{64}}$ ← Estimate.

Using ASET, Estimate is $\frac{31}{64}$ with error at most $\frac{1}{1792} < \frac{1}{1000}$ as desired

37. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+2}}{3^n} = \frac{2^2}{3^0} - \frac{2^3}{3^1} + \frac{2^4}{3^2} - \dots$

Geometric $\quad a=4 \quad r = \frac{-2}{3}$ $\quad$ SUM $= \frac{a}{1-r} = \frac{4}{1-(-2/3)} = \frac{4}{5/3} = \boxed{\frac{12}{5}}$

OR get to $\sum x^n = \frac{1}{1-x}$ Form

$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+2}}{3^n} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n \cdot 2^2}{3^n} = 4 \sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{3^n} = 4 \sum_{n=0}^{\infty} \left(\frac{-2}{3}\right)^n$$

$$= 4\left[\frac{1}{1-(-2/3)}\right] = 4\left(\frac{1}{5/3}\right) = 4 \cdot 3/5 = \boxed{\frac{12}{5}}$$

38. $1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots = e^1 = \boxed{e}$

39. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!} = \cos\pi = \boxed{-1}$

40. $\sum_{n=0}^{\infty} \frac{(-1)^n 49^n \pi^{2n}}{4^n (2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 7^{2n} \pi^{2n}}{2^{2n} (2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{7\pi}{2}\right)^{2n}}{(2n+1)!} \cdot \frac{\left(\frac{7\pi}{2}\right)}{\left(\frac{7\pi}{2}\right)}$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{7\pi}{2}\right)^{2n+1}}{(2n+1)!} \cdot \frac{1}{\left(\frac{7\pi}{2}\right)} = \frac{2}{7\pi} \cancel{\sin\frac{7\pi}{2}} = \boxed{\frac{-2}{7\pi}}$$

extra $\quad -1$

41. $\sum_{n=0}^{\infty} \frac{(-9)^n \pi^{2n+1}}{4^n (2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} \pi^{2n} \cdot \pi}{2^{2n} (2n)!} = \pi \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{3\pi}{2}\right)^{2n}}{(2n)!}$ extra

$$= \pi \cancel{\cos\left(\frac{3\pi}{2}\right)} = \boxed{0}$$

0

42. $\sum_{n=0}^{\infty} \frac{(-\pi^2)^n}{36^n (2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\pi/6\right)^{2n}}{(2n)!} = \cos\left(\pi/6\right) = \boxed{\frac{\sqrt{3}}{2}}$

43. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{7n+1}}{n!} = x\sum_{n=0}^{\infty} \frac{x^{7n}}{n!} = x\sum_{n=0}^{\infty} \frac{(x^7)^n}{n!} = \boxed{xe^{x^7}}$

44. $1 - \frac{1}{2} + \frac{1}{2^2 \cdot 2!} - \frac{1}{2^3 \cdot 3!} + \frac{1}{2^4 \cdot 4!} - \dots$ $\qquad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$= 1 - \frac{1}{2} + \frac{(\frac{1}{2})^2}{2!} - \frac{(\frac{1}{2})^3}{3!} + \frac{(\frac{1}{2})^4}{4!} - \dots$

$= 1 + (-\frac{1}{2}) + \frac{(-\frac{1}{2})^2}{2!} + \frac{(-\frac{1}{2})^3}{3!} + \dots = \boxed{e^{-1/2}}$

45. $-\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$ looks like $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$

$\arctan 1 = 1 - \frac{1}{3} + \frac{1}{5} - \dots$

$= \arctan(1) - 1$

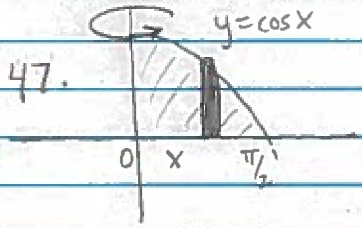
but missing first term when $n=0$

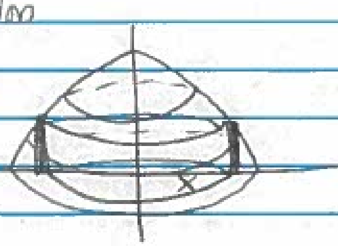
$= \boxed{\frac{\pi}{4} - 1}$

46. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}(n+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{1}{2})^{n+1}}{n+1} = \ln(1+\tfrac{1}{2}) = \boxed{\ln\left(\tfrac{3}{2}\right)}$

Volumes of Revolution

47.



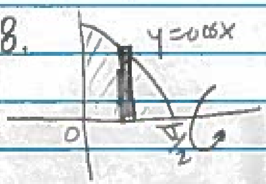


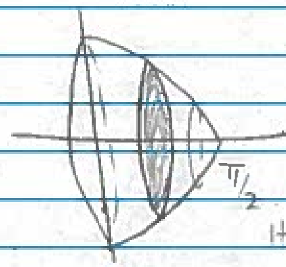
$$V=\int_0^{\pi/2} 2\pi \text{ radius}\cdot\text{height}\,dx = 2\pi\int_0^{\pi/2} x\cos x\,dx = 2\pi\left[x\sin x\Big|_0^{\pi/2} - \underbrace{\int_0^{\pi/2}\sin x\,dx}_{+\cos x\,\big|_0^{\pi/2}}\right]$$

$$\boxed{\begin{array}{ll} u=x & dv=\cos x\,dx \\ du=dx & v=\sin x \end{array}}$$

$$=2\pi\left[\left(\tfrac{\pi}{2}\sin\tfrac{\pi}{2} - 0\sin 0\right)+\left(\cos\tfrac{\pi}{2}-\cos 0\right)\right] = 2\pi\left[\tfrac{\pi}{2}-1\right] = \boxed{\pi^2-2\pi}$$

48.



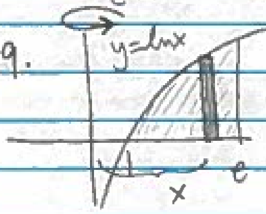


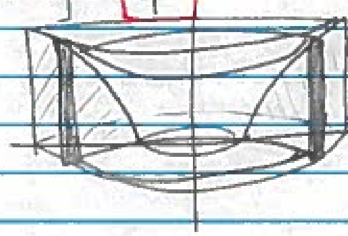
Half angle Identity

$$V=\int_0^{\pi/2}\pi\,\text{radius}^2\,dx = \pi\int_0^{\pi/2}\cos^2 x\,dx = \pi\int_0^{\pi/2}\tfrac{1}{2}\left[1+\cos(2x)\right]dx = \tfrac{\pi}{2}\left[x+\frac{\sin(2x)}{2}\right]\Big|_0^{\pi/2}$$

$$=\tfrac{\pi}{2}\left[\left(\tfrac{\pi}{2}+\frac{\sin\pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right] = \boxed{\frac{\pi^2}{4}}$$

49.





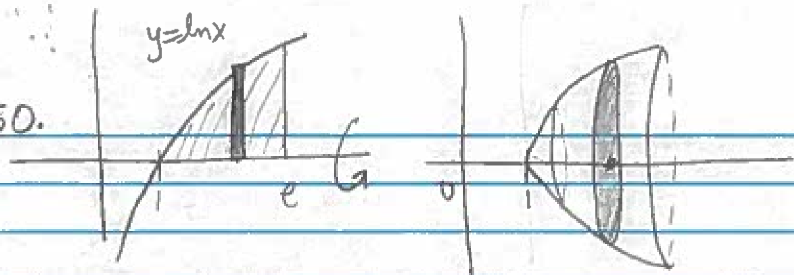
$$V=\int_1^e 2\pi\text{ radius}\cdot\text{height}\,dx = 2\pi\int_1^e x\ln x\,dx = 2\pi\left[(\ln x)\frac{x^2}{2}\Big|_1^e - \int_1^e \frac{x^2}{2}\left(\frac{1}{x}\right)dx\right]$$

$$\boxed{\begin{array}{ll} u=\ln x & dv=x\,dx \\ du=\frac{1}{x}dx & v=\frac{x^2}{2} \end{array}}$$

$$=2\pi\left[(\ln x)\frac{x^2}{2}\Big|_1^e - \int_1^e\frac{x}{2}\,dx\right] = 2\pi\left[(\ln x)\frac{x^2}{2}\Big|_1^e - \frac{x^2}{4}\Big|_1^e\right] = 2\pi\left[(\ln e)\frac{e^2}{2} - (\ln 1)\frac{1}{2} - \left(\frac{e^2}{4}-\frac{1}{4}\right)\right]$$

$$=2\pi\left[\underset{\frac{2e^2}{4}}{\frac{e^2}{2}} - \frac{e^2}{4}+\frac{1}{4}\right] = 2\pi\left[\frac{e^2}{4}+\frac{1}{4}\right] = \boxed{\frac{\pi}{2}\left(e^2+1\right)}$$

50.

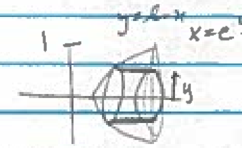


Double I.B.P.

$$V=\int_1^e \pi\,\text{radius}^2\,dx=\pi\int_1^e (\ln x)^2\,dx=\pi\left[(\ln x)^2x\Big|_1^e-\int_1^e x\cdot\frac{2\ln x}{x}\,dx\right]=\pi\left[(\ln x)^2x\Big|_1^e-2\left(x\ln x\Big|_1^e-\int_1^e x\cdot\frac{1}{x}\,dx\right)\right]$$

$$\boxed{\begin{array}{ll} u=(\ln x)^2 & dv=dx \\ du=\frac{2\ln x}{x}dx & v=x \end{array}} \qquad \boxed{\begin{array}{ll} u=\ln x & dv=dx \\ du=\frac{1}{x}dx & v=x \end{array}}$$

$$=\pi\left[(\ln x)^2x\Big|_1^e-2x\ln x\Big|_1^e+2x\Big|_1^e\right]=\pi\left[\left((\ln e)^2e-(\ln 1)^2 1\right)-2\left(e\ln e-\ln 1\right)+2(e-1)\right]$$

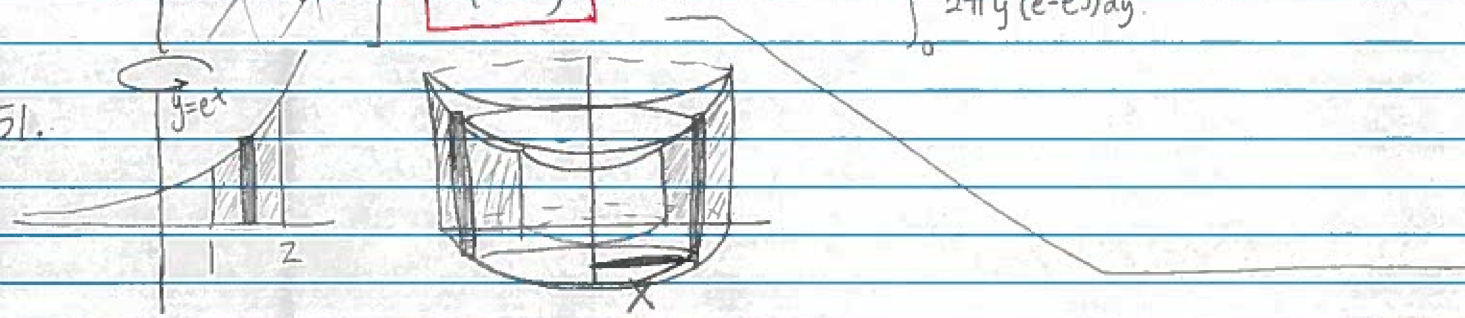


$$=\pi\left[e-\cancel{2e}+\cancel{2e}-2\right]=\boxed{\pi(e-2)}$$

Shells would be

$$\int_0^1 2\pi y\,(e-e^y)\,dy.$$

51.

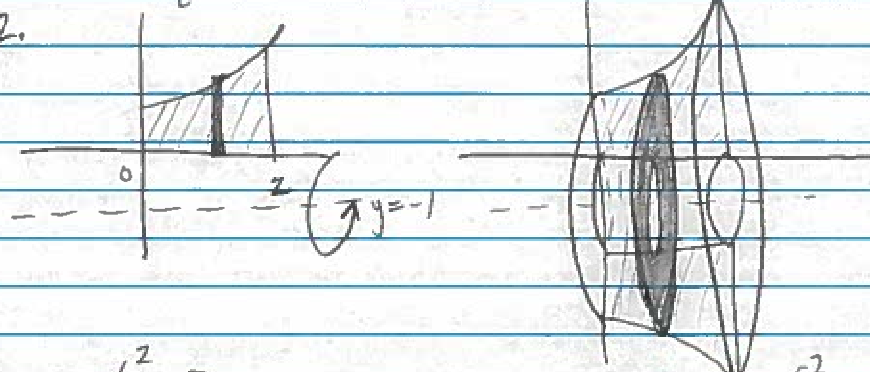


$$V=\int_1^2 2\pi\,\text{radius}\cdot\text{height}\,dx=2\pi\int_1^2 xe^x\,dx=2\pi\left[xe^x\Big|_1^2-\int_1^2 e^x\,dx\right]=2\pi\left[xe^x\Big|_1^2-e^x\Big|_1^2\right]$$

$$\boxed{\begin{array}{ll} u=x & dv=e^x dx \\ du=dx & v=e^x \end{array}}$$

$$=2\pi\left[(2e^2-e^1)-(e^2-e^1)\right]=2\pi\left[2e^2-\cancel{e^1}-e^2+\cancel{e^1}\right]=2\pi\left[e^2\right]=\boxed{2\pi e^2}$$

52.

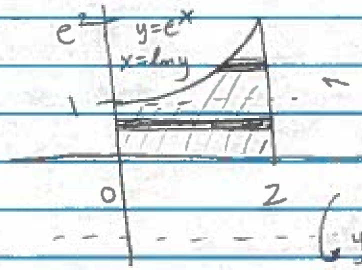


$$V=\int_0^2 \pi\left[(\text{outer radius})^2-(\text{inner radius})^2\right]dx=\pi\int_0^2 (e^x+1)^2-1^2\,dx$$

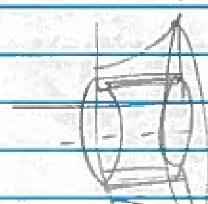
52 (continued)

$$= \pi \int_0^2 e^{2x} + 2e^x + \cancel{1} - \cancel{1}\, dx = \pi \int_0^2 e^{2x} + 2e^x\, dx = \pi \left[\frac{e^{2x}}{2} + 2e^x \right] \Big|_0^2$$

$$= \pi \left[\left(\frac{e^4}{2} + 2e^2 \right) - \underbrace{\left(\frac{e^0}{2} + 2e^0 \right)}_{5/2} \right] = \boxed{\pi \left[\frac{e^4}{2} + 2e^2 - 5/2 \right]}$$



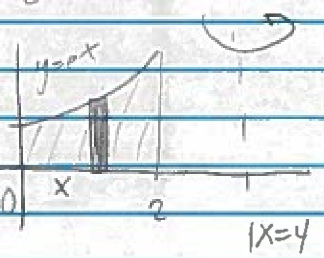
← if use cylinders, will get different height functions for $0 \le y \le 1$ and $1 \le y \le e^2$

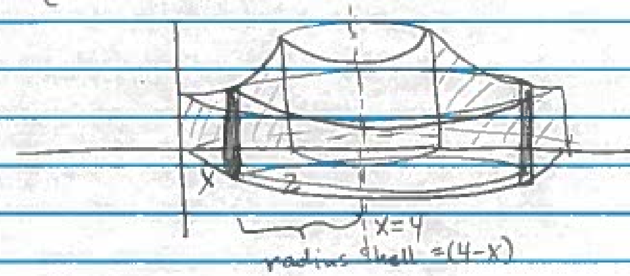


Cylinders

$$V = \int_0^{e^2} 2\pi\, \text{radius} \cdot \text{height}\, dy = 2\pi \left[\int_0^1 y(2)\, dy + \int_1^{e^2} y(2 - \ln y)\, dy \right]$$

53.





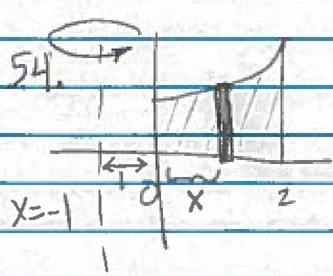
$$V = \int_0^2 2\pi\, \text{radius} \cdot \text{height}\, dx = 2\pi \int_0^2 (4-x)e^x\, dx = 2\pi \int_0^2 4e^x - xe^x\, dx = 2\pi \left[4e^x \Big|_0^2 - \left(xe^x \Big|_0^2 - \int_0^2 e^x\, dx \right) \right]$$

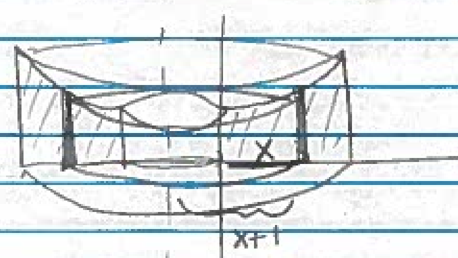
I.B.P. $u = x$ $dv = e^x dx$, $du = dx$ $v = e^x$

$+e^x \Big|_0^2$

$$= 2\pi \left[(4e^2 - 4e^0) - (2e^2 - 0e^0) + (e^2 - e^0) \right] = 2\pi \left[4e^2 - 4 - 2e^2 + e^2 - 1 \right] = \boxed{2\pi \left[3e^2 - 5 \right]}$$

54.





if we used washers, we would have 2 different radius formulas for $0 \le y \le 1$ and $1 \le y \le e$

$$V = \int_0^2 2\pi\, \text{radius} \cdot \text{height}\, dx = 2\pi \int_0^2 (x+1)e^x\, dx = 2\pi \left[(x+1)e^x \Big|_0^2 - \int_0^2 e^x\, dx \right]$$

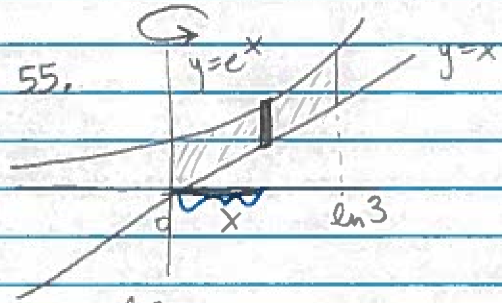
I.B.P. $u = x+1$ $dv = e^x dx$, $du = dx$ $v = e^x$

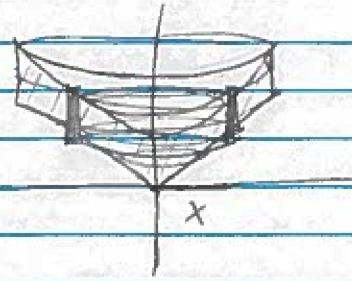
54.(continued)

$$= 2\pi\left[(x+1)e^x\Big|_0^2 - e^x\Big|_0^2\right] = 2\pi\left[xe^x + \cancel{e^x} - \cancel{e^x}\Big|_0^2\right] = 2\pi\left(xe^x\right)\Big|_0^2 = 2\pi\left[2e^2 - \cancel{0e^0}\right] = \boxed{4\pi e^2}$$

Some cancelling

55.





Note: Disks/Washer Slicing would involve one piece with Disks for radius info and top chunk with Washers.

$$V = \int_0^{\ln 3} 2\pi \,\text{radius} \cdot \text{height}\, dx = 2\pi \int_0^{\ln 3} x\left(e^x - x\right)dx = 2\pi \int_0^{\ln 3} xe^x - x^2\, dx$$

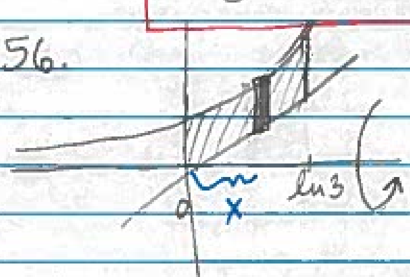
I.B.P. piece $\boxed{\begin{matrix} u = x & dv = e^x dx \\ du = dx & v = e^x \end{matrix}}$

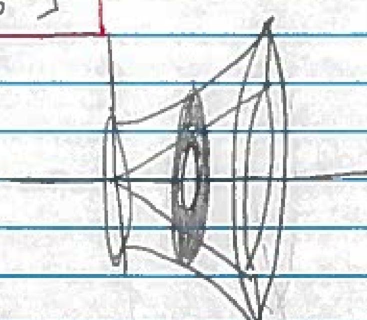
$$= 2\pi\left[xe^x\Big|_0^{\ln 3} - \int_0^{\ln 3} e^x dx - \frac{x^3}{3}\Big|_0^{\ln 3}\right] = 2\pi\left[xe^x\Big|_0^{\ln 3} - e^x\Big|_0^{\ln 3} - \frac{x^3}{3}\Big|_0^{\ln 3}\right]$$

$$= 2\pi\left[\left(\ln 3\, e^{\ln 3} - 0\right) - \left(e^{\ln 3} - e^0\right) - \left(\frac{(\ln 3)^3}{3} - 0\right)\right] = 2\pi\left[\underbrace{3\ln 3}_{\ln 27} - 3 + 1 - \frac{(\ln 3)^3}{3}\right]$$

$$= \boxed{2\pi\left[3\ln 3 - 2 - \frac{(\ln 3)^3}{3}\right]}$$

56.



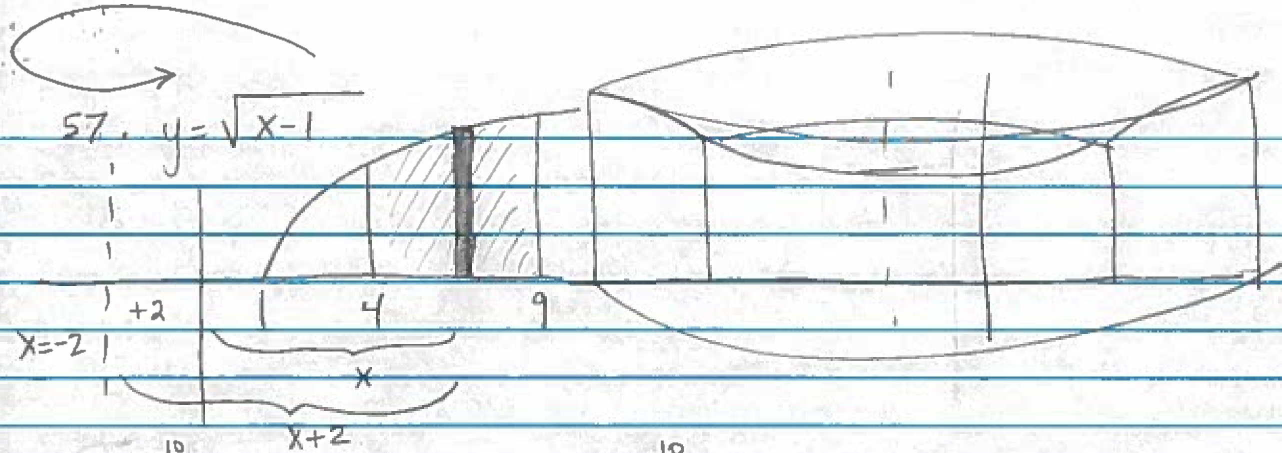


cylinders would involve to different Height functions for 2 Different Regions

$$V = \int_0^{\ln 3} \pi\left[(\text{outer radius})^2 - (\text{inner radius})^2\right]dx = \pi\int_0^{\ln 3}\left[\left(e^x\right)^2 - x^2\right]dx = \pi\int_0^{\ln 3} e^{2x} - x^2\, dx$$

$$= \pi\left[\frac{e^{2x}}{2} - \frac{x^3}{3}\right]_0^{\ln 3} = \pi\left[\left(\frac{e^{2\ln 3}}{2} - \frac{(\ln 3)^3}{3}\right) - \left(\frac{e^0}{2} - 0\right)\right] = \pi\left[\frac{9}{2} - \frac{(\ln 3)^3}{3} - \frac{1}{2}\right] = \pi\left[\frac{8}{2} - \frac{(\ln 3)^3}{3}\right] = \boxed{\pi\left[4 - \frac{(\ln 3)^3}{3}\right]}$$

57. $y=\sqrt{x-1}$



$$V=\int_5^{10} 2\pi\,\text{radius}\cdot\text{height}\,dx = 2\pi\int_5^{10}(x+2)\sqrt{x-1}\;dx$$

inverted u-sub.

$\boxed{\begin{matrix}u=x-1\\ du=dx\end{matrix}} \Rightarrow x=u+1 \Rightarrow x+2=u+3$ $\quad\boxed{\begin{matrix}x=5\Rightarrow u=4\\ x=10\Rightarrow u=9\end{matrix}}$

Factor out 2

$$=2\pi\int_{u=4}^{u=9}(u+3)\sqrt{u}\;du = 2\pi\int_4^9 u^{3/2}+3u^{1/2}\;du = 2\pi\left[\frac{2}{5}u^{5/2}+\cancel{3}\,\frac{2}{\cancel{3}}u^{3/2}\right]_4^9$$

$$=4\pi\left[\left(\frac{9^{5/2}}{5}+9^{3/2}\right)-\left(\frac{4^{5/2}}{5}+4^{3/2}\right)\right]$$

$$=4\pi\left[\frac{3^5}{5}+3^3-\frac{2^5}{5}-2^3\right]$$

$$=4\pi\left[\frac{243}{5}+27-\frac{32}{5}-8\right]$$

$$=4\pi\left[\frac{211}{5}+19\right]$$

$$=4\pi\left[\frac{211}{5}+\frac{95}{5}\right]$$

$$=4\pi\left[\frac{306}{5}\right]$$

$$=\boxed{\frac{1224}{5}\pi}$$

Be careful using $\sqrt{a^2} = |a|$

Parametric Equations

58. $x = t + \frac{1}{t}$ $y = 2\ln t$ $1 \le t \le 3$ Point $(\frac{5}{2}, 2\ln 2)$ corresponds to Parameter $t = 2$.

a. $\frac{dx}{dt} = 1 - \frac{1}{t^2}$ $\frac{dy}{dt} = \frac{2}{t}$ $y = 2\ln t = 2\ln 2$

$\Rightarrow t = 2$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2}{t}}{1-1/t^2}$$

Point Slope Form

$$y - 2\ln 2 = 4/3\,(x - 5/2)$$
$$= \frac{4}{3}x - \frac{10}{3}$$
$$\boxed{y = 4/3\,x - 10/3 + 2\ln 2}$$

Slope $\left.\frac{dy}{dx}\right|_{t=2} = \frac{\frac{2}{2}}{1-1/4} = \frac{1}{3/4} = \frac{4}{3}$

b. $$L = \int_1^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_1^3 \sqrt{\left(1 - \frac{1}{t^2}\right)^2 + \left(\frac{2}{t}\right)^2}\, dt = \int_1^3 \sqrt{1 - \frac{2}{t^2} + \frac{1}{t^4} + \frac{4}{t^2}}\, dt$$

$$= \int_1^3 \sqrt{1 + \frac{2}{t^2} + \frac{1}{t^4}}\, dt = \int_1^3 \sqrt{\left(1 + \frac{1}{t^2}\right)^2}\, dt = \int_1^3 \underbrace{1 + \frac{1}{t^2}}_{\text{Note } \oplus}\, dt$$

t^{-2}

$$= t + \left.\frac{t^{-1}}{-1}\right|_1^3 = \left(3 - \frac{1}{3}\right) - \cancel{(1-1)} = \frac{9}{3} - \frac{1}{3} = \boxed{\frac{8}{3}}$$

59. $x = \tan t - t$ $y = \ln(\cos t)$ $0 \le t \le \pi/3$ $\tan \pi/6 = \frac{\sin \pi/6}{\cos \pi/6} = \frac{\frac{1}{2}}{\sqrt{3}/2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\frac{dx}{dt} = \sec^2 t - 1$ $\frac{dy}{dt} = \frac{1}{\cos t}(-\sin t) = -\tan t$

$= \tan^2 t$

a. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\tan t}{\tan^2 t} = \frac{-1}{\tan t}$ $\left.\frac{dy}{dx}\right|_{t=\pi/6} = \frac{-1}{\tan \pi/6} = \frac{-1}{1/\sqrt{3}} = \boxed{-\sqrt{3}}$

b. $$L = \int_0^{\pi/3} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{\pi/3} \sqrt{(\tan^2 t)^2 + (-\tan t)^2}\, dt = \int_0^{\pi/3} \sqrt{\tan^4 t + \tan^2 t}\, dt$$

$$= \int_0^{\pi/3} \sqrt{\tan^2 t\underbrace{(\tan^2 t + 1)}_{\sec^2 t}}\, dt = \int_0^{\pi/3} \underbrace{\tan t \sec t}_{\text{Both } \tan t \text{ and } \sec t \ \oplus \text{ on } (0, \pi/3)}\, dt = \sec t\Big|_0^{\pi/3} = \underbrace{\sec \pi/3}_{\frac{1}{\cos \pi/3} = 2} - \underbrace{\sec 0}_{\frac{1}{\cos 0} = 1} = 2 - 1 = \boxed{1}$$

6a. $x = t - e^{t}$ $y = 1 - 4e^{t/2}$ $0 \le t \le \ln 5$

$\frac{dx}{dt} = 1 - e^{t}$ $\frac{dy}{dt} = -4e^{t/2}\left(\frac{1}{2}\right) = -2e^{t/2}$

a. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2e^{t/2}}{1-e^{t}}$

$$\frac{dy}{dx}\Big|_{t=\ln 4} = \frac{-2e^{\frac{\ln 4}{2}}}{1-\underbrace{e^{\ln 4}}_{4}} = \frac{-2e^{\ln 4^{1/2}}}{-3} = \frac{-4}{-3} = \boxed{\frac{4}{3}}$$

b. $$L = \int_0^{\ln 5}\sqrt{(1-e^{t})^2 + (-2e^{t/2})^2}\,dt = \int_0^{\ln 5}\sqrt{1-2e^{t}+(e^{t})^2+4e^{t}}\,dt = \int_0^{\ln 5}\sqrt{1+2e^{t}+(e^{t})^2}\,dt$$

$$= \int_0^{\ln 5}\sqrt{(1+e^{t})^2}\,dt = \int_0^{\ln 5}\underbrace{1+e^{t}}_{\oplus\text{ here}}\,dt = t+e^{t}\Big|_0^{\ln 5} = (\ln 5 + e^{\ln 5}) - (0 + e^{0}) = \boxed{\ln 5 + 4}$$

c. x-axis

$$SA = \int_0^{\ln 5} 2\pi y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 2\pi\int_0^{\ln 5}(1-4e^{t/2})\underbrace{\sqrt{(1-e^{t})^2+(-2e^{t/2})^2}}_{(1+e^{t})^2}\,dt$$

already computed this as Perfect Square in Part b

$$= \boxed{2\pi\int_0^{\ln 5}(1-4e^{t/2})(1+e^{t})\,dt} \quad \text{OR} \quad = \boxed{2\pi\int_0^{\ln 5} 1 + e^{t} - 4e^{t/2} - 4e^{3/2\,t}\,dt}$$ Can STOP here

$-4e^{1/2\,t}e^{t}$

ignore! computation.

$$\cancel{= 2\pi\left[t + e^{t} - 4\cdot 2e^{t/2} - 4\cdot\tfrac{2}{3}e^{3/2\,t}\right]_0^{\ln 5} = 2\pi\left[\left(\ln 5 + e^{\ln 5} - 8e^{\frac{\ln 5}{2}} - \tfrac{8}{3}e^{\frac{3}{2}\ln 5}\right) - \left(0 + e^{0} - 8e^{0} - \tfrac{8}{3}e^{0}\right)\right]}$$

$$\cancel{= 2\pi\left[\ln 5 + 5 - 8\sqrt{5} - \tfrac{8}{3}\cdot 5^{3/2} - 1 + 8 + \tfrac{8}{3}\right]} = \boxed{2\pi\left[\ln 5 \underbrace{- 8\sqrt{5} - \tfrac{8}{3}\cdot 5^{3/2}}_{-\frac{64\sqrt{5}}{3}} \underbrace{+ 12 + \tfrac{8}{3}}_{44/3}\right]}$$ Blah!

$$= \boxed{2\pi\left[\ln 5 - \frac{64\sqrt{5}}{3} + \frac{44}{3}\right]}$$

61. $x = e^{t}\cos t$ $y = e^{t}\sin t$

$\frac{dx}{dt} = e^{t}(-\sin t) + \cos t\, e^{t}$ $\frac{dy}{dt} = e^{t}\cos t + \sin t\, e^{t}$

$$L = \int_0^{\ln\pi}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^{\ln\pi}\sqrt{(-e^{t}\sin t + e^{t}\cos t)^2 + (e^{t}\cos t + e^{t}\sin t)^2}\,dt$$

expand algebra $e^{2t}\sin^2 t \cancel{- 2e^{2t}\sin t\cos t} + e^{2t}\cos^2 t + e^{2t}\cos^2 t \cancel{+ 2e^{2t}\cos t\sin t} + e^{2t}\sin^2 t$ →

cancel!

(continued)

61.(continued) $= \int_0^{\ln\pi} \sqrt{2e^{2t}\sin^2 t + 2e^{2t}\cos^2 t}\; dt = \int_0^{\ln\pi} \sqrt{2e^{2t}(\cancel{\sin^2 t + \cos^2 t})}\; dt$ $\;\; 1$

$= \sqrt{2}\int_0^{\ln\pi} \underbrace{e^t}_{\oplus \text{ always}}\, dt = \sqrt{2}\, e^t\Big|_0^{\ln\pi} = \sqrt{2}\left[\cancel{e^{\ln\pi}}^{\,\pi} - \cancel{e^0}^{\,1}\right] = \boxed{\sqrt{2}\,[\pi - 1]}$

62. $x = 3t^2 \quad y = 2t^3 \qquad 0 \le t \le \ln 3$ Point $(3,2)$ corresponds to $t=1$

$x = 3t^2 = 3$ or $y = 2t^3 = 2$

$\frac{dx}{dt} = 6t \quad \frac{dy}{dt} = 6t^2$

a. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t^2}{6t} = t$

Point Slope Form

$y - 2 = 1(x - 3)$

$= x - 3$

$\boxed{y = x - 1}$

$\frac{dy}{dx}\Big|_{t=1} = 1$

b. $L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt = \int_0^1 \sqrt{36t^2 + 36t^4}\, dt = \int_0^1 \sqrt{36t^2(1+t^2)}\, dt$

$= \int_0^1 \underset{\substack{\uparrow \\ \oplus \\ \text{here}}}{6t} \sqrt{1+t^2}\, dt = 3\int_1^2 \sqrt{u}\, du = 3 \cdot \frac{2}{3} u^{3/2}\Big|_1^2 = \boxed{2[\underbrace{(2)^{3/2}}_{2\sqrt{2}} - 1]} = \boxed{4\sqrt{2} - 2}$

$\boxed{\begin{array}{l} u = 1+t^2 \\ du = 2t\,dt \\ \frac{1}{2}du = t\,dt \end{array}}$ $\boxed{\begin{array}{l} t=0 \Rightarrow u=1 \\ t=1 \Rightarrow u=2 \end{array}}$ change variables

y-axis

Clarify! $\boxed{\text{Do S.A. for } 0 \le t \le 1}$

same piece as in b.

c. $SA = \int_0^1 2\pi x(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 2\pi \int_0^1 3t^2 \sqrt{(6t)^2 + (6t^2)^2}\, dt = 2\pi \int_0^1 3t^2 \cdot 6t\sqrt{1+t^2}\, dt$

$= 36\pi \int_0^1 \overset{t^2 \cdot t}{t^3} \sqrt{1+t^2}\, dt = \frac{36\pi}{2} \int_1^2 (u-1)\sqrt{u}\, du = 18\pi \int_1^2 u^{3/2} - u^{1/2}\, du = 18\pi \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^2$ factor out 2.

$\boxed{\begin{array}{l} u = 1+t^2 \\ du = 2t\,dt \\ \frac{1}{2}du = t\,dt \end{array}}$ $\Rightarrow t^2 = u - 1$ $\boxed{\begin{array}{l} t=0 \Rightarrow u=1 \\ t=1 \Rightarrow u=2 \end{array}}$ $\boxed{= 36\pi\left[\left(\frac{\overbrace{2^{5/2}}^{4\sqrt{2}}}{5} - \frac{\overbrace{(2)^{3/2}}^{2\sqrt{2}}}{3}\right) - \left(\frac{1}{5} - \frac{1}{3}\right)\right]}$ Simplify...

$36\pi\left[\left(\frac{4\sqrt{2}-1}{5}\right) - \left(\frac{2\sqrt{2}-1}{3}\right)\right] = 36\pi\left[\frac{12\sqrt{2} - 3 - 10\sqrt{2} + 5}{15}\right] = 36\pi\left(\frac{2\sqrt{2}+2}{15}\right) = \boxed{\frac{24}{5}(1+\sqrt{2})\pi}$

63. $x=\sin^3 t \quad y=\cos^3 t \quad 0\le t\le \pi/2$

Point $\left(\frac{3\sqrt{3}}{8}, \frac{1}{8}\right)$ corresponds to $t=\pi/3$

$\frac{dx}{dt} = 3\sin^2 t\cdot\cos t \quad \frac{dy}{dt} = 3\cos^2 t(-\sin t)$

$y=\cos^3 t = 1/8 \Rightarrow \cos t = 1/2 \Rightarrow t=\pi/3$

check $x=\sin^3 t = \frac{3\sqrt{3}}{8} = \frac{(\sqrt{3})^3}{8} \Rightarrow \sin t = \frac{\sqrt{3}}{2}$, $t=\pi/3$

a. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\cancel{3}\cos^{\cancel{2}} t\sin t}{\cancel{3}\sin^{\cancel{2}} t\cos t} = \frac{-\cos t}{\sin t}$

$\frac{dy}{dx}\Big|_{t=\pi/3} = \frac{-\cos \pi/3}{\sin \pi/3} = \frac{-1/2}{\sqrt{3}/2}\cdot\frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$

Point Slope Form

$y-\frac{1}{8} = -\frac{1}{\sqrt{3}}\left(x-\frac{3\sqrt{3}}{8}\right)$

$\boxed{y=-\frac{x}{\sqrt{3}}+\frac{1}{2}}$

b. $L=\int_0^{\pi/2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt = \int_0^{\pi/2}\sqrt{(3\sin^2 t\cos t)^2+(-3\cos^2 t\sin t)^2}\,dt = \int_0^{\pi/2}\sqrt{9\sin^4 t\cos^2 t+9\cos^4 t\sin^2 t}\,dt$

$= \int_0^{\pi/2}\sqrt{9\sin^2 t\cos^2 t(\underbrace{\sin^2 t+\cos^2 t}_{1})}\,dt \quad \int_0^{\pi/2} \underbrace{3\sin t\cos t}_{\textcircled{*}\text{ here}}\,dt = 3\,\frac{\sin^2 t}{2}\Big|_0^{\pi/2} = \frac{3}{2}\left(\sin^2\frac{\pi}{2} - \sin^2 0\right) = \boxed{\frac{3}{2}}$

x-axis

use original given parameter bounds

c. $SA = \int_0^{\pi/2} 2\pi\, y(t)\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt = 2\pi\int_0^{\pi/2}\cos^3 t\,\underbrace{\sqrt{(3\sin^2 t\cos t)^2+(-3\cos^2 t\sin t)^2}}_{\text{already computed in b}}\,dt$

$=2\pi\int_0^{\pi/2}\cos^3 t\,[3\sin t\cos t]\,dt = 6\pi\int_0^{\pi/2}\cos^4 t\sin t\,dt = -6\pi\int_1^0 u^4\,du = -\frac{6\pi u^5}{5}\Big|_1^0$

$\boxed{\begin{array}{l}u=\cos t\\ du=-\sin t\,dt\\ -du=\sin t\,dt\end{array}} \quad \boxed{\begin{array}{l}t=0\Rightarrow u=1\\ t=\pi/2\Rightarrow u=\cos\pi/2=0\end{array}}$

$= \frac{-6\pi}{5}[0-1] = \boxed{\frac{6\pi}{5}}$

64. $x = 3-2t \qquad y = e^t + e^{-t}$

$\frac{dx}{dt} = -2 \qquad \frac{dy}{dt} = e^t - e^{-t}$

a. $L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^1 \sqrt{(-2)^2 + (e^t - e^{-t})^2}\, dt = \int_0^1 \sqrt{4 + e^{2t} - 2e^{\cancel{t-t}\,0} + e^{-2t}}\, dt$

$= \int_0^1 \sqrt{e^{2t} + 2 + e^{-2t}}\, dt = \int_0^1 \sqrt{(e^t + e^{-t})^2}\, dt = \int_0^1 \underbrace{e^t + e^{-t}}_{\oplus \text{ always.}}\, dt = e^t - e^{-t}\Big|_0^1$

$= (e - e^{-1}) - \cancel{(e^0 - e^0)} = \boxed{e - \frac{1}{e}}$

x-axis

b. $S.A = \int_0^1 2\pi y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 2\pi \int_0^1 (e^t + e^{-t}) \underbrace{\sqrt{(-2)^2 + (e^t - e^{-t})^2}}_{\text{already computed in a.}}\, dt$

$= 2\pi \int_0^1 (e^t + e^{-t}) \sqrt{(e^t + e^{-t})^2}\, dt = 2\pi \int_0^1 (e^t + e^{-t})(e^t + e^{-t})\, dt = 2\pi \int_0^1 e^{2t} + 2e^0 + e^{-2t}\, dt$

$= 2\pi \left[\frac{e^{2t}}{2} + 2t - \frac{e^{-2t}}{2} \right]_0^1 = 2\pi \left[\left(\frac{e^2}{2} + 2 - \frac{e^{-2}}{2} \right) - \left(\cancel{\frac{e^0}{2}} + 0 - \cancel{\frac{e^0}{2}} \right) \right] = \boxed{\pi e^2 + 4\pi - \pi e^{-2}}$

cancel

y-axis

c. $S.A = \int_0^1 2\pi x(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 2\pi \int_0^1 (3-2t) \underbrace{\sqrt{(-2)^2 + (e^t - e^{-t})^2}}_{\text{already computed in a}}\, dt$

$= 2\pi \int_0^1 \underbrace{(3-2t)(e^t + e^{-t})\, dt}_{\text{would involve 2 Integration By Parts}} = \cdots$