Math 121 　　Midterm Exam #3 　　December 6, 2017

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

- Numerical answers such as $\sin\left(\dfrac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, $\arctan\sqrt{3}$ or $\cosh(\ln 3)$ should be simplified.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [20 Points]　Find the **Interval** and **Radius** of Convergence for each of the following power series. Analyze carefully and with full justification.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,(3x-4)^n}{5^n\,\sqrt{n}}$ 　　　　(b) $\displaystyle\sum_{n=1}^{\infty} n!\,(x-6)^n$ 　　　　(c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{(2n)!}$

2. [12 Points]　Find the **MacLaurin series** representation for each of the following functions. **State** the Radius of Convergence for each series. Your answer should be in sigma notation. Simplify.

(a) $f(x) = x^3 \ln(1+2x)$ 　　　　　　(b) $f(x) = 1 - \cos\left(x^2\right)$

3. [10 Points]　Use the MacLaurin Series representation for 　$f(x) = x\sin\left(x^2\right)$ 　to

$$\textbf{Estimate}\quad \int_0^1 x\sin\left(x^2\right)\ dx \quad\text{with error less than }\ \frac{1}{10}.$$

Justify in words that your error is indeed less than $\dfrac{1}{10}$.

4. [8 Points]　Estimate $\arctan\left(\dfrac{1}{2}\right)$ with error less than $\dfrac{1}{100}$. Justify.

5. [20 Points]　Find the **sum** for each of the following series. Simplify, if possible.

(a) $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} - \ldots$ 　　　(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\,\pi^{2n}}{36^n\,(2n+1)!}$ 　　　(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\,(\ln 27)^n}{3^{n+1}\,n!}$

(d) $1 - e + \dfrac{e^2}{2!} - \dfrac{e^3}{3!} + \dfrac{e^4}{4!} - \dfrac{e^5}{5!} + \ldots$ (e) $\dfrac{1}{e} - \dfrac{1}{2e^2} + \dfrac{1}{3e^3} - \dfrac{1}{4e^4} + \dfrac{1}{5e^5} - \ldots$ (f) $-\dfrac{\pi^2}{2!} + \dfrac{\pi^4}{4!} - \dfrac{\pi^6}{6!} + \dfrac{\pi^8}{8!} - \ldots$

6. [18 Points]   Volumes of Revolution

~~(a) Consider the region bounded by $y = \arctan x$, $y = \dfrac{\pi}{4}$, and $x = 0$. Rotate this region about the horizontal line $\boxed{y = 1}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Washer Method. Sketch the solid, along with one of the approximating washers.~~

~~(b) Consider the region bounded by $y = \ln x$, $y = 2$, and $x = 10$. Rotate this region about the vertical line $\boxed{x = 1}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.~~

~~(c) Consider the region bounded by $y = 2 + e^x$, $y = \cos x$, $x = 0$ and $x = \dfrac{\pi}{2}$. Rotate this region about the vertical line $\boxed{x = 5}$. Set-up, **BUT DO NOT EVALUATE!!**, the integral to compute the volume of the resulting solid using the Cylindrical Shells Method. Sketch the solid, along with one of the approximating shells.~~

7. [12 Points]   Parametric Equations

Consider the Parametric Curve given by $x = \ln t + \ln(1 - t^2)$ and $y = \sqrt{8} \ \arcsin t$.

<u>COMPUTE</u> the **Arclength** of this parametric curve for $\dfrac{1}{4} \le t \le \dfrac{1}{2}$. **SHOW** that the answer

simplifies to $\boxed{\ln\left(\dfrac{5}{2}\right)}$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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~~**OPTIONAL BONUS #1**   Compute $\displaystyle\sum_{n=0}^{\infty} \frac{n^3 (\ln 3)^n}{n!}$~~

OPTIONAL BONUS #2   Compute $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n}{(2n+1)\ 3^n}$

OPTIONAL BONUS #3   Find the sum of the Power Series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{(n+3)!}$

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