Math 121 Midterm Exam #2 April 8-11, 2021

Due Sunday, April 11, in Gradescope by 11:59 pm ET

• This is an *Open Notes* Exam. You can use materials, homeworks problems, lecture notes, etc. that you manually worked on.

• There is **NO** *Open Internet* allowed. You can only access our Main Course Webpage.

• You are not allowed to work on or discuss these problems with anyone, including the Professor or Math Fellow TA.

• Submit your final work in Gradescope in the Exam 2 entry.

• Please *show* all of your work and *justify* all of your answers. No Calculators.

1. [25 Points] Compute the following **Improper** integrals. Simplify all answers. Justify your work.

(a) $\displaystyle\int_e^{\infty} \frac{\ln x}{x^3}\, dx$

(b) $\displaystyle\int_{-2}^{5} \frac{8}{x^2 - 4x - 12}\, dx$

(c) $\displaystyle\int_{-\infty}^{\infty} \frac{x^4}{4 + x^{10}}\, dx$

2. [8 Points] Show that the sequence $\left\{ \left(\frac{n}{n+1}\right)^n \right\}_{n=1}^{\infty}$ Converges to $\frac{1}{e}$. Justify.

3. [15 Points] Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2 + 4n + 7}$. Demonstrate **two different** methods to show that this series Diverges.

(a) First, you must use the Integral Test. You can **SKIP** checking the 3 preconditions here.

(b) Second, use a different method. Your choice.

4. [26 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any convergence test(s) you use, and justify all of your work.

(a) $\sum_{n=1}^{\infty} n^8 + 8$ (b) $\sum_{n=1}^{\infty} \frac{n^8+8}{n^8+1}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n^8+1}$

(d) $\sum_{n=8}^{\infty} \frac{n^8}{\ln n}$ (e) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^8}$

(f) $\sum_{n=1}^{\infty} 8$ (g) $\sum_{n=1}^{\infty} \frac{1}{8^n}$ (h) $\sum_{n=1}^{\infty} \left(\frac{1}{8} + \frac{1}{8^n}\right)$

5. [6 Points] Consider the Series $\sum_{n=1}^{\infty} \frac{8n^2+n}{n^8}$. Show this series Converges by **splitting** it into the sum of two series that are each Convergent. Justify all steps.

6. [3 Points] Use the result from #5 above to explain why $\sum_{n=1}^{\infty} (-1)^n \frac{8n^2+n}{n^8}$ Converges.

For 7, 8, and 9, determine whether the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Divergent**. Name any convergence test(s) you use, and justify all of your work.

7. [3 Points] $\sum_{n=1}^{\infty} (-1)^n \frac{8n^2+n}{n^8}$ Feel free to reference (not repeat) your work above.

8. [7 Points] $\sum_{n=1}^{\infty} \frac{(-1)^n}{8n+3}$

9. [7 Points] $\sum_{n=1}^{\infty} \frac{(-1)^n\,(3n)!\,n^8}{8^n\,n^n\,(n!)^2}$