Review Packet for Exam #1

Fall 2020, Math 121, D. Benedetto

Derivatives: Compute the derivative for each of the following functions. Do not worry about simplifying your answers:

1. $f(x) = \arcsin x \cdot \arctan x + \arctan(\sin(\ln x))$

2. $f(x) = \dfrac{\sinh(x^2 - 2)}{x + \sin^{-1} x}$

3. $f(x) = \sin\left(e^{\arcsin e^x}\right)$

4. $f(x) = \ln\left(1 - \arcsin\left(\dfrac{7}{x}\right)\right)$

5. Compute $f''(x)$ where $f(x) = \ln(1 - x)$.

6. Compute $f''(x)$ where $f(x) = \arctan(3x)$.

7. Compute $f''(x)$ where $f(x) = \arcsin(4x)$.

Proofs:

8. Prove that $\dfrac{d}{dx} \arcsin x = \dfrac{1}{\sqrt{1 - x^2}}$

9. Prove that $\dfrac{d}{dx} \arctan x = \dfrac{1}{1 + x^2}$

10. Prove that $\dfrac{d}{dx} \sinh^{-1} x = \dfrac{1}{\sqrt{1 + x^2}}$

11. Prove that $\dfrac{d}{dx} \arcsin(5x) = \dfrac{5}{\sqrt{1 - 25x^2}}$

12. Prove that $\dfrac{d}{dx} \ln x = \dfrac{1}{x}$

13. Prove that $\dfrac{d}{dx} \arctan(3x) = \dfrac{3}{1 + 9x^2}$

14. Prove, using Integration, that $\displaystyle\int \dfrac{1}{9 + x^2}\, dx = \dfrac{1}{3} \arctan\left(\dfrac{x}{3}\right) + C$

15. Prove, using Integration, that $\displaystyle\int \dfrac{1}{\sqrt{16 - x^2}}\, dx = \arcsin\left(\dfrac{x}{4}\right) + C$

Limits: Compute each of the following limit.

16. $\lim\limits_{x\to\infty} \left(1 + \dfrac{1}{x}\right)^x$

17. $\lim\limits_{x\to 0} \dfrac{7xe^x - \arctan(7x)}{\sinh x + \ln(1-x)}$

18. $\lim\limits_{x\to 0} \dfrac{\cosh(4x) - 1 - \arctan(4x) + 4x}{\ln(1-x) + \arcsin x}$

19. $\lim\limits_{x\to 0} \dfrac{1 - e^{-4x} - \arctan(4x)}{x^2}$

20. $\lim\limits_{x\to 0^+} x^3 \ln x$

21. $\lim\limits_{x\to 0^+} x \ln\left(\dfrac{1}{x}\right)$

22. $\lim\limits_{x\to\infty} x^{\frac{1}{x^2}}$

23. $\lim\limits_{x\to 0^+} (1 - 2x)^{\frac{1}{x}}$

24. $\lim\limits_{x\to\infty} \left(x^3 + 1\right)^{\frac{1}{\ln x}}$

25. $\lim\limits_{x\to\infty} x^2 \sin\left(\dfrac{1}{x^2}\right)$

26. $\lim\limits_{x\to 0^+} \sqrt{x} \ln x$

27. $\lim\limits_{x\to\infty} \left(1 - \dfrac{3}{x}\right)^{4x}$

28. $\lim\limits_{x\to\infty} \left(1 + \ln\left(1 - \dfrac{6}{x^2}\right)\right)^{x^2}$

29. $\lim\limits_{x\to\infty} \left(1 - \arctan\left(\dfrac{5}{x^4}\right)\right)^{3x^4}$

30. CHALLENGE: $\lim\limits_{x\to\infty} \left(\sqrt{1 - \dfrac{5}{x^2}} - \sinh\left(\dfrac{1}{x^2}\right)\right)^{5x^2}$

31. $\lim\limits_{x\to\infty} \left(e^{\frac{1}{x^8}} - \dfrac{8}{x^8}\right)^{x^8}$

32. $\lim\limits_{x\to\infty} \left(\dfrac{x}{x+3}\right)^x$

Integrals: Compute each of the following integrals.

33. $\int \left(e^x + x\right)^2 \, dx$

34. $\int x \sin^2 x \, dx$

35. $\int \dfrac{1}{\sqrt{25 - x^2}} \, dx$

36. $\int \dfrac{1}{x^2 + 25} \, dx$

37. $\int \dfrac{1}{x\sqrt{9 - \ln^2 x}} \, dx$

38. $\int x \arcsin x \, dx$

39. $\int \dfrac{1}{(4 - x^2)^{\frac{3}{2}}} \, dx$

40. $\int_1^e \ln x \, dx$

41. $\int \dfrac{\ln(2x^5)}{x^2} \, dx$

42. $\int \cos^5 x \, dx$

43. $\int \ln^2\left(x^{20}\right) \, dx$

44. $\int \sin^5 x \cos^2 x \, dx$

45. $\int \sin^2 x \cos^3 x \, dx$

46. $\int_{e^{\sqrt{3}}}^{e^3} \dfrac{1}{x\left(9 + (\ln x)^2\right)} \, dx$

47. $\int_1^e \dfrac{1}{x\left(1 + (\ln x)^2\right)^{\frac{3}{2}}} \, dx$

48. $\int_0^{\frac{\pi}{2}} \dfrac{\cos x}{\left(1 + \sin^2 x\right)^{\frac{7}{2}}} \, dx$

49. $\int_1^{\sqrt{3}} \dfrac{1}{\sqrt{4 - x^2}} \, dx$

50. $\int \dfrac{1}{(x^2 + 4)^2} \, dx$

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51. $\displaystyle\int \frac{1}{(x^2+4)^{\frac{7}{2}}} \; dx$

52. $\displaystyle\int_{-1}^{0} x^4 \arcsin x \; dx$

53. $\displaystyle\int \frac{\sinh x}{\sqrt{16 - \cosh^2 x}} \; dx$

54. $\displaystyle\int_{0}^{1} x \tan^{-1}(x^2) \; dx$

55. $\displaystyle\int \frac{x^2}{x^6 + 1} \; dx$

56. $\displaystyle\int_{1}^{e^2} x \ln \sqrt{x} \; dx$

57. $\displaystyle\int \frac{x^2}{(1 - x^2)^{\frac{3}{2}}} \; dx$

58. $\displaystyle\int_{1}^{e} (\ln x)^2 \; dx$

59. $\displaystyle\int_{0}^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}} + \frac{1}{x^2 + 9} \; dx$

60. $\displaystyle\int \frac{x^2}{\sqrt{16 - x^2}} \; dx$

61. $\displaystyle\int x^3 \sqrt{9 - x^2} \; dx$

62. $\displaystyle\int \frac{x^2}{x^2 + 3} \; dx$

63. $\displaystyle\int_{-3}^{3} \sqrt{9 - x^2} \; dx$

64. $\displaystyle\int_{1}^{e} \sqrt{x} \ln x \; dx$

65. $\displaystyle\int \frac{x + 3}{\sqrt{4 - x^2}} \; dx$

66. CHALLENGE: $\displaystyle\int (\arcsin x)^2 \; dx$