

## Approximations using only a few terms of the MacLaurin Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

$$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \frac{1}{9!} + \frac{1}{(10)!} + \frac{1}{(11)!} + \frac{1}{(12)!} + \dots$$

$$e = 2.71828182845904523536028747135\dots$$

$$e \approx 1 + 1 = 2$$

$$e \approx 1 + 1 + \frac{1}{2!} = 2.5$$

$$e \approx 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} = 2.708333\dots$$

$$e \approx 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \frac{1}{9!} + \frac{1}{(10)!} = 2.7182818011\dots$$

$$e \approx 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \frac{1}{9!} + \frac{1}{(10)!} + \dots + \frac{1}{(19)!} + \frac{1}{(20)!} = 2.71828182845904523533978449\dots$$

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\ldots$$

$$e^{\frac{1}{2}}=1+\frac{1}{2}+\frac{\left(\frac{1}{2}\right)^2}{2!}+\frac{\left(\frac{1}{2}\right)^3}{3!}+\frac{\left(\frac{1}{2}\right)^4}{4!}+\ldots$$

$$\sqrt{e}=1.648721270700128146\ldots$$

$$e^{\frac{1}{2}} \approx 1 + \frac{1}{2} = 1.5$$

$$e^{\frac{1}{2}} \approx 1 + \frac{1}{2} + \frac{\left(\frac{1}{2}\right)^2}{2!} = 1.625$$

$$e^{\frac{1}{2}} \approx 1 + \frac{1}{2} + \frac{\left(\frac{1}{2}\right)^2}{2!} + \frac{\left(\frac{1}{2}\right)^3}{3!} = 1.645833\overline{3}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$$

$$\sin 1 = 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} + \dots$$

$$\sin 1 = 0.841570984807\dots$$

$$\sin 1 \approx 1$$

$$\sin 1 \approx 1 - \frac{1}{3!} = \frac{5}{6} = 0.8\overline{333} \quad \text{with error at most } \frac{1}{5!} = \frac{1}{120} = 0.008\overline{33} \quad \text{by ASET}$$

$$\sin 1 \approx 1 - \frac{1}{3!} + \frac{1}{5!} = \frac{101}{120} = 0.841\overline{666} \quad \text{with error at most } \frac{1}{7!} = \frac{1}{5040} = 0.00019841269\dots \quad \text{by ASET}$$

$$\sin 1 \approx 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} = \frac{4241}{5040} = 0.84146825\dots \quad \text{with error at most } \frac{1}{9!} = \frac{1}{362880} = 0.0000027557\dots \\ \text{by ASET}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$$

$$\sin 0.16 = 0.16 - \frac{(0.16)^3}{3!} + \frac{(0.16)^5}{5!} - \frac{(0.16)^7}{7!} + \frac{(0.16)^9}{9!} + \dots$$

$$\sin 0.16 = 0.159318206614245963311463\dots$$

$$\sin 0.16 \approx 0.16$$

$$\sin 0.16 \approx 0.16 - \frac{(0.16)^3}{3!} = 0.15931733\dots \quad \text{with error at most } \frac{(0.16)^5}{5!} = 0.000000873\dots \quad \text{by ASET}$$

$$\sin 0.16 \approx 0.16 - \frac{(0.16)^3}{3!} + \frac{(0.16)^5}{5!} = 0.1593182071466\dots$$

$$\text{with error at most } \frac{(0.16)^7}{7!} \approx 5.326 * 10^{-10} \quad \text{by ASET}$$