Math 121 Final Exam May 11, 2020

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\ln(e^7)$, $e^{-\ln 5}$, or $\arctan(\sqrt{3})$, should be simplified.
- Please *show* all of your work and *justify* all of your answers.
- You may work for no more than 2 hours. You MUST be in front of the ZOOM camera.
- When done, immediately upload to the Math 121 Final Exam entry in Gradescope. **TAG** problems.

1. [20 Points] Evaluate the following **integrals**.

(a) Show that $\displaystyle\int_0^1 x^2 \arcsin x \, dx = \frac{\pi}{6} - \frac{2}{9}$

(b) $\displaystyle\int \frac{1}{(x^2+4)^2} \, dx \overset{\text{hint}}{=} \int \frac{1}{\left(\sqrt{x^2+4}\right)^4} \, dx$

2. [20 Points] For each of the following **improper integrals**, determine whether it converges or diverges. If it converges, find its value. Simplify.

(a) $\displaystyle\int_{-\infty}^{5} \frac{1}{x^2-4x+7} \, dx$

(b) $\displaystyle\int_0^1 \ln x \, dx$

3. [24 Points] Find the **sum** of each of the following series (which do converge). Simplify.

(a) $1 - e + \dfrac{e^2}{2!} - \dfrac{e^3}{3!} + \dfrac{e^4}{4!} - \dfrac{e^5}{5!} + \dots$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{8}{5^n}$

(c) $-\dfrac{1}{2e^2} + \dfrac{1}{3e^3} - \dfrac{1}{4e^4} + \dfrac{1}{5e^5} - \dots$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \, \pi^{2n}}{9^n \, (2n+1)!}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \, \pi^{2n+1}}{9 \, (2n)!}$

(f) $4 + 4 - \dfrac{4}{3} + \dfrac{4}{5} - \dfrac{4}{7} + \dfrac{4}{9} - \dots$

4. [30 Points] For each part, you do not need to choose complicated Infinite Series. If you choose to use the Ratio Test, then you can only use the Ratio Test for **at most** one part (a), (b), or (c).

(a) Give an example of an **Alternating** Series which is **Absolutely Convergent**. You **cannot** choose just $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ or just a Geometric Series. Continue on to **PROVE** that this series is Absolutely Convergent.

(b) Give an example of an Infinite Series which is **Divergent**. You **cannot** choose just $\displaystyle\sum_{n=1}^{\infty}$ constant or a p-series or a Geometric Series. Continue on to **PROVE** that this series is Divergent.

(c) Give an example of an **Alternating** Series which is **Conditionally Convergent**. You **cannot** choose just $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$. Continue on to **PROVE** that this series is Conditionally Convergent.

5. [16 Points] Find the **Interval** and **Radius** of Convergence for the following power series

$$\sum_{n=1}^{\infty} n^n \cdot \ln n \cdot (x-6)^n$$

Analyze carefully and with full justification.

6. [10 Points] Please analyze with detail and justify carefully. Simplify.

Use MacLaurin Series to **Estimate** $\arctan\left(\frac{1}{2}\right)$ with error less than $\frac{1}{100}$.

7. [20 Points] For both parts, you do **not** need to find the Radius of Convergence. Your answer should be in Sigma notation $\sum_{n=0}^{\infty}$.

(a) Use MacLaurin Series to compute $\int x^4 e^{-x^3}\, dx$.

(b) Use MacLaurin Series to compute $\frac{d}{dx}\left[x^3 \sin(6x)\right]$.

8. [20 Points] For **each** of the following problems, do the following **THREE** things:

1. **Sketch** the Polar curve(s) and **shade** the described bounded region.
2. Set-Up but **DO NOT EVALUATE** an Integral representing the area of the described bounded region.
3. Set-Up but **DO NOT EVALUATE** another **slightly different** Integral representing the same area of the described bounded region.

(a) The **area** bounded outside the polar curve $r = 2 + 2\sin\theta$ and inside $r = 6\sin\theta$.

(b) The **area** that lies inside both of the curves $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.