Homework 11 Final Answers

Section 11.1

5. $\boxed{\frac{1}{5}, -\frac{1}{25}, \frac{1}{125}, -\frac{1}{625}, \frac{1}{3125}, \ldots}$

7. $\boxed{\frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \frac{1}{720}, \ldots}$

17. $\boxed{a_n = \frac{(-1)^{n+1} n^2}{n+1}}$ or $\frac{(-1)^{n-1} n^2}{n+1}$ or $\frac{(-1)^{n+3} n^2}{n+1}$ ...

23. $\boxed{5}$

37. $\boxed{0}$

40. $\boxed{0}$

41. $\boxed{0}$

45. $\boxed{1}$

47. $\boxed{e^2}$

49. $\boxed{\ln 2}$

50. $\boxed{0}$

73. Explain why decreasing → show $f'(x) < 0$?!

To show Bounded, show there is an upper bound and lower bound

↳ See Below for Book's Definitions

10 Definition A sequence $\{a_n\}$ is called **increasing** if $a_n < a_{n+1}$ for all $n \geqslant 1$, that is, $a_1 < a_2 < a_3 < \cdots$. It is called **decreasing** if $a_n > a_{n+1}$ for all $n \geqslant 1$. A sequence is **monotonic** if it is either increasing or decreasing.

SOLUTION 2 Consider the function $f(x) = \dfrac{x}{x^2 + 1}$:

$$f'(x) = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2} < 0 \qquad \text{whenever } x^2 > 1$$

Thus f is decreasing on $(1, \infty)$ and so $f(n) > f(n + 1)$. Therefore $\{a_n\}$ is decreasing. ■

11 Definition A sequence $\{a_n\}$ is **bounded above** if there is a number M such that

$$a_n \leqslant M \qquad \text{for all } n \geqslant 1$$

It is **bounded below** if there is a number m such that

$$m \leqslant a_n \qquad \text{for all } n \geqslant 1$$

If it is bounded above and below, then $\{a_n\}$ is a **bounded sequence**.

For instance, the sequence $a_n = n$ is bounded below $(a_n > 0)$ but not above. The sequence $a_n = n/(n + 1)$ is bounded because $0 < a_n < 1$ for all n.

We know that not every bounded sequence is convergent [for instance, the sequence $a_n = (-1)^n$ satisfies $-1 \leqslant a_n \leqslant 1$ but is divergent from Example 7] and not every