

Homework #9

Due **Wednesday, October 8th** in Gradescope by 11:59 pm ET

Goal: Exploring Improper Integrals, for both Type I (unbounded domain) and Type II (unbounded range). We will need IBP, Complete the Square, Partial Fractions, and some u-sub here. We may also need L'Hôpital's Rule to finish a few of the limits at hand.

FIRST: Read through and understand the following three Examples.

Ex:

$$\begin{aligned}
 \int_{-\infty}^7 \frac{1}{x^2 - 6x + 25} dx &= \lim_{t \rightarrow -\infty} \int_t^7 \frac{1}{x^2 - 6x + 25} dx \stackrel{\text{complete square}}{=} \lim_{t \rightarrow -\infty} \int_t^7 \frac{1}{(x-3)^2 + 16} dx \\
 &= \lim_{t \rightarrow -\infty} \int_{t-3}^4 \frac{1}{u^2 + 16} du = \lim_{t \rightarrow -\infty} \frac{1}{4} \arctan\left(\frac{u}{4}\right) \Big|_{t-3}^4 = \lim_{t \rightarrow -\infty} \frac{1}{4} \left(\arctan\left(\frac{4}{4}\right) - \arctan\left(\frac{t-3}{4}\right) \right) \\
 &= \lim_{t \rightarrow -\infty} \frac{1}{4} \left(\arctan(1) - \arctan\left(\frac{t-3}{4}\right) \right) = \frac{1}{4} \left(\frac{\pi}{4} - \left(-\frac{\pi}{2}\right) \right) = \frac{1}{4} \left(\frac{3\pi}{4} \right) = \boxed{\frac{3\pi}{16}}
 \end{aligned}$$

$ \begin{aligned} u &= x - 3 \\ du &= dx \end{aligned} $	$ \begin{aligned} x = t &\Rightarrow u = t - 3 \\ x = 7 &\Rightarrow u = 7 - 3 = 4 \end{aligned} $
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Ex:

$$\begin{aligned}
 \int_0^1 \frac{\ln x}{\sqrt{x}} dx &= \lim_{s \rightarrow 0^+} \int_s^1 \frac{\ln x}{\sqrt{x}} dx = \lim_{s \rightarrow 0^+} \int_s^1 (\ln x) x^{-\frac{1}{2}} dx = \lim_{s \rightarrow 0^+} 2\sqrt{x} \ln x \Big|_s^1 - 2 \int_s^1 \frac{1}{\sqrt{x}} dx \\
 &= \lim_{s \rightarrow 0^+} 2\sqrt{x} \ln x \Big|_s^1 - 4\sqrt{x} \Big|_s^1 = \lim_{s \rightarrow 0^+} 2\sqrt{1} \ln 1 - 4\sqrt{1} - \left(2\sqrt{s} \ln s^{0 \cdot (-\infty)} - 4\sqrt{s} \right) \\
 &\stackrel{*}{=} \boxed{-4}
 \end{aligned}$$

IBP

$ \begin{aligned} u &= \ln x & dv &= x^{-\frac{1}{2}} dx \\ du &= \frac{1}{x} dx & v &= 2\sqrt{x} \end{aligned} $
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* L'H Finish: $\lim_{s \rightarrow 0^+} \sqrt{s} \ln s \stackrel{0 \cdot (-\infty)}{=} \lim_{s \rightarrow 0^+} \frac{\ln s}{\frac{1}{\sqrt{s}}} \stackrel{\text{L'H}}{=} \lim_{s \rightarrow 0^+} \frac{\frac{1}{s}}{-\frac{1}{2s^{\frac{3}{2}}}} = \lim_{s \rightarrow 0^+} -2\sqrt{s} = 0$

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Ex:

$$\begin{aligned}
 \int_0^6 \frac{8}{x^2 - 4x - 12} dx &= \int_0^6 \frac{8}{(x-6)(x+2)} dx = \lim_{t \rightarrow 6^-} \int_0^t \frac{8}{(x-6)(x+2)} dx \\
 &\stackrel{\text{PFD}}{=} \lim_{t \rightarrow 6^-} \int_0^t \frac{1}{x-6} - \frac{1}{x+2} dx = \lim_{t \rightarrow 6^-} \ln|x-6| - \ln|x+2| \Big|_0^t \\
 &= \lim_{t \rightarrow 6^-} \ln|t-6|^{-\infty} - \ln|t+2|^{0+} - (\ln|-6| - \ln 2) \\
 &= -\infty - \ln 8 - \ln 6 + \ln 2 = \boxed{-\infty} \text{ Diverges}
 \end{aligned}$$

PFD:

$$\frac{8}{(x-6)(x+2)} = \frac{A}{x-6} + \frac{B}{x+2}$$

Clearing the denominator yields:

$$8 = A(x+2) + B(x-6)$$

$$8 = Ax + 2A + Bx - 6B$$

$$8 = (A+B)x + (2A-6B)$$

$$\text{so that } A+B=0 \text{ and } 2A-6B=8$$

$$\text{Solve for } A=1 \text{ and } B=-1$$

Compute each of the following Integrals. Simplify when possible.

$$1. \int_{-\infty}^0 \frac{1}{3-4x} dx$$

$$2. \int_1^{\infty} \frac{1}{(2x+1)^3} dx$$

$$3. \int_2^{\infty} \frac{x}{e^{3x}} dx$$

$$4. \int_e^{\infty} \frac{\ln x}{x^3} dx$$

$$5. \int_e^{\infty} \frac{1}{x(\ln x)^2} dx$$

$$6. \int_e^{\infty} \frac{1}{x \ln x} dx$$

$$7. \int_{-\infty}^7 \frac{1}{x^2 - 4x + 29} dx$$

$$8. \int_0^5 \frac{6}{x^2 - 4x - 5} dx$$

$$9. \int_0^{e^5} \frac{1}{x [25 + (\ln x)^2]} dx$$

$$10. \int_1^2 \frac{1}{x \ln x} dx$$

$$11. \int_0^1 x \ln x dx$$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30–9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00–3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: 10–11:30 am

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

You are welcome at Office Hours all the time. *Please come!*