

Homework #8Due **Friday, October 3rd** in Gradescope by 11:59 pm ET

Goal: Exploring Integrals using Secant Trig Substitutions, some Complete the Square algebra and also Partial Fractions Decomposition (Proper cases).

FIRST: Read through and understand the following two Examples.

Recall: IF the denominator FACTORS, go use PFD; otherwise try Complete the Square algebra to start

Ex:

$$\begin{aligned} \int \frac{6}{x^2 + 2x + 8} dx &\stackrel{\text{complete square}}{=} \int \frac{6}{(x+1)^2 + 7} dx = 6 \int \frac{1}{u^2 + 7} du \\ &= 6 \left(\frac{1}{\sqrt{7}} \right) \arctan \left(\frac{u}{\sqrt{7}} \right) + C = \boxed{\frac{6}{\sqrt{7}} \arctan \left(\frac{x+1}{\sqrt{7}} \right) + C} \end{aligned}$$

$$\begin{aligned} u &= x + 1 \\ du &= dx \end{aligned}$$

Ex:

$$\begin{aligned} \int_{-3}^1 \frac{6}{x^2 + 2x - 8} dx &\stackrel{\text{factor}}{=} \int_{-3}^1 \frac{6}{(x-2)(x+4)} dx \stackrel{\text{PFD}}{=} \int_{-3}^1 \frac{1}{x-2} - \frac{1}{x+4} dx \\ &= \ln |x-2| - \ln |x+4| \Big|_{-3}^1 = \ln |-1| - \ln 5 - (\ln |-5| - \ln 1) \\ &= 0 - \ln 5 - \ln 5 + 0 = \boxed{-2 \ln 5} \end{aligned}$$

Partial Fractions Decomposition:

$$\frac{6}{(x-2)(x+4)} = \frac{A}{x-2} + \frac{B}{x+4}$$

Clearing the denominator yields:

$$6 = A(x+4) + B(x-2)$$

$$6 = Ax + 4A + Bx - 2B$$

$$6 = (A+B)x + (4A-2B)$$

$$\text{so that } A+B=0 \text{ and } 4A-2B=6$$

$$\text{Solve for } A=1 \text{ and } B=-1$$

Continue to NEXT Page for HW problems.

Compute each of the following Integrals. Simplify when possible.

1. $\int \frac{\sqrt{x^2 - 9}}{x^3} dx$

2. $\int \frac{1}{x^2 \sqrt{x^2 - 16}} dx$

$\uparrow$ sub $\boxed{\begin{array}{l} x = 3 \sec \theta \\ dx = 3 \sec \theta \tan \theta d\theta \end{array}}$

$\uparrow$ sub $\boxed{\begin{array}{l} x = 4 \sec \theta \\ dx = 4 \sec \theta \tan \theta d\theta \end{array}}$

$\uparrow$ hint: $\frac{\tan^2 \theta}{\sec^2 \theta} = \sin^2 \theta$ why?

3. $\int \frac{1}{\sqrt{4 - 4x - x^2}} dx$

4. $\int_{-1}^1 \frac{1}{x^2 + 4x + 7} dx$

5. $\int \sqrt{3 - 2x - x^2} dx$

6. $\int_3^5 \frac{6}{x^2 - 4x + 7} dx$

7. $\int_0^3 \frac{6}{x^2 - 4x - 5} dx$

8. $\int_0^1 \frac{x - 4}{x^2 - 5x + 6} dx$

9. $\int \frac{10}{(x - 1)(x^2 + 9)} dx$

$\uparrow$ use PFD $\frac{10}{(x - 1)(x^2 + 9)} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 9}$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30–9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00–3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: 10–11:30 am

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

Please e-mail with questions/concerns: dbenedetto@amherst.edu

Either *Start early* or *don't start late*.

Thanks for working hard. I appreciate it!