

Homework #7Due **Friday, September 26th** in Gradescope by 11:59 pm**Goal:** Exploring Trigonometric Integrals and Trigonometric Substitution**FIRST:** Read through and understand the following two Examples.

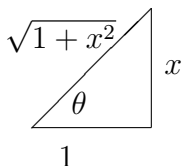
Ex:

$$\begin{aligned}
\int \frac{1}{[1+x^2]^{\frac{7}{2}}} dx &= \int \frac{1}{(1+\tan^2 \theta)^{\frac{7}{2}}} \cdot \sec^2 \theta d\theta = \int \frac{1}{(\sec^2 \theta)^{\frac{7}{2}}} \cdot \sec^2 \theta d\theta \\
&= \int \frac{1}{(\sqrt{\sec^2 \theta})^7} \cdot \sec^2 \theta d\theta = \int \frac{1}{(\sec \theta)^7} \cdot \sec^2 \theta d\theta \\
&= \int \frac{\sec^2 \theta}{\sec^7 \theta} d\theta = \int \frac{1}{\sec^5 \theta} d\theta \\
&= \int \cos^5 \theta d\theta = \int \cos^4 \theta \cos \theta d\theta \\
&= \int (1 - \sin^2 \theta)^2 \cos \theta d\theta = \int (1 - w^2)^2 dw \\
&= \int 1 - 2w^2 + w^4 dw = w - \frac{2w^3}{3} + \frac{w^5}{5} + C \\
&= \sin \theta - \frac{2 \sin^3 \theta}{3} + \frac{\sin^5 \theta}{5} + C \\
&= \boxed{\frac{x}{\sqrt{1+x^2}} - \frac{2}{3} \left(\frac{x}{\sqrt{1+x^2}} \right)^3 + \frac{1}{5} \left(\frac{x}{\sqrt{1+x^2}} \right)^5 + C}
\end{aligned}$$

Trig. Sub

$$x = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$



$$w = \sin \theta$$

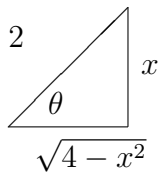
$$dw = \cos \theta d\theta$$

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Ex:

$$\begin{aligned}
 \int \frac{x^2}{\sqrt{4-x^2}} dx &= \int \frac{(2 \sin \theta)^2}{\sqrt{4-4 \sin^2 \theta}} 2 \cos \theta d\theta = \int \frac{4 \sin^2 \theta}{\sqrt{4(1-\sin^2 \theta)}} 2 \cos \theta d\theta \\
 &= 4 \int \frac{\sin^2 \theta}{\sqrt{4} \sqrt{\cos^2 \theta}} 2 \cos \theta d\theta = 4 \int \frac{\sin^2 \theta}{2 \cos \theta} 2 \cos \theta d\theta \\
 &= 4 \int \sin^2 \theta d\theta = 4 \int \frac{1 - \cos(2\theta)}{2} d\theta \\
 &= 2 \int 1 - \cos(2\theta) d\theta = 2 \left(\theta - \frac{\sin(2\theta)}{2} \right) + C \\
 &= 2 \left(\theta - \frac{2 \sin \theta \cos \theta}{2} \right) + C = 2 (\theta - \sin \theta \cos \theta) + C \\
 &= \boxed{2 \left[\arcsin \left(\frac{x}{2} \right) - \left(\frac{x}{2} \right) \left(\frac{\sqrt{4-x^2}}{2} \right) \right] + C}
 \end{aligned}$$

Trig. Substitute $\boxed{\begin{array}{l} x = 2 \sin \theta \\ dx = 2 \cos \theta d\theta \end{array}}$



Compute each of the following Integrals. Simplify.

$$1. \int \sin^2 x \cos^3 x dx \quad 2. \int_0^{\frac{\pi}{2}} \sin^5 x dx \quad 3. \int_0^{\frac{\pi}{2}} \cos^2 \theta d\theta$$

$$4. \int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x dx \quad 5. \int x \sin^2 x dx \quad 6. \int_0^1 x^3 \sqrt{1-x^2} dx \quad \text{use Trig Sub}$$

$$7. \int \sqrt{9-x^2} dx \quad 8. \int \frac{1}{(4+x^2)^{\frac{5}{2}}} dx \quad 9. \int x \arcsin x dx$$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30–9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00–3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: 10–11:30 am

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

This is the end of exam #1 material

Go to office hours and also you're welcome at Math Fellow TA hours