Math 121, Section(s) 01 and 02, Fall 2025

Homework #6

Due **Wednesday, September 24th** in Gradescope by 11:59 pm

Goal: Exploring Limits using L'Hôpital's Rule, and Integrals using Integration By Parts

FIRST: Read through and understand the following two Examples.

Ex: $\lim_{x\to\infty}\left(e^{\frac{1}{x}}-\frac{4}{x}\right)^x \overset{1^\infty}{=} \lim_{x\to\infty} e^{\ln\left(\left(e^{\frac{1}{x}}-\frac{4}{x}\right)^x\right)} = e^{\lim_{x\to\infty}\ln\left(\left(e^{\frac{1}{x}}-\frac{4}{x}\right)^x\right)} = e^{\lim_{x\to\infty} x\ln\left(e^{\frac{1}{x}}-\frac{4}{x}\right)}$

$\overset{\infty\cdot 0}{=} e^{\lim_{x\to\infty}\frac{\ln\left(e^{\frac{1}{x}}-\frac{4}{x}\right)}{\frac{1}{x}}} \overset{\left(\frac{0}{0}\right)^{\text{L'H}}}{=} e^{\lim_{x\to\infty}\frac{\left(\frac{1}{e^{\frac{1}{x}}-\frac{4}{x}}\right)\cdot\left[e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)+\frac{4}{x^2}\right]}{-\frac{1}{x^2}}}$

$= e^{\lim_{x\to\infty}\left(\frac{1}{e^{\frac{1}{x}}-\frac{4}{x}}\right)\cdot\left[e^{\frac{1}{x}}\left(-\frac{1}{\cancel{x^2}}\right)+\frac{4}{\cancel{x^2}}\right](-\cancel{x^2})} = e^{\lim_{x\to\infty}\left(\frac{1}{e^{\frac{1}{x}}-\frac{4}{x}}\right)\cdot\left(e^{\frac{1}{x}}(1)-4\right)} = \boxed{e^{-3}}$

Ex: $\int \arctan\left(\frac{1}{x}\right)\,dx = x\arctan\left(\frac{1}{x}\right) \overset{--}{+} \int \frac{x}{x^2+1}\,dx = x\arctan\left(\frac{1}{x}\right)+\frac{1}{2}\int\frac{1}{w}\,dw$

$= x\arctan\left(\frac{1}{x}\right)+\frac{1}{2}\ln|w|+C = \boxed{x\arctan\left(\frac{1}{x}\right)+\frac{\ln|x^2+1|}{2}+C}$

IBP

$u = \arctan\left(\frac{1}{x}\right)$ $\qquad dv = 1dx$

$du = \frac{1}{1+\left(\frac{1}{x}\right)^2}\cdot\left(-\frac{1}{x^2}\right)dx$ $\qquad v = x$

$du = -\frac{1}{x^2+1}dx$ $\qquad \leftarrow$ simplify

w-sub

$w = x^2+1$

$dw = 2x\,dx$

$\frac{1}{2}dw = x\,dx$

Continue to NEXT Page for HW problems.

Compute each of the following Limits. Simplify.

1. $\lim\limits_{x\to\infty} \dfrac{\ln\left(5+e^{3x}\right)}{x}$

2. $\lim\limits_{x\to\infty} \left(\dfrac{x}{x+1}\right)^{x}$ likely need Quotient Rule in part of Chain Rule for $\ln\left(\dfrac{x}{x+1}\right)$

3. $\lim\limits_{x\to\infty} \left(e^{\frac{1}{x^6}} - \dfrac{6}{x^6}\right)^{x^6}$ Reference Sample Problem above

Compute each of the following Integrals using Integration by Parts. Simplify.

4. $\displaystyle\int x\cos(5x)\ dx$

5. $\displaystyle\int_0^1 \arctan x\ dx$

6. $\displaystyle\int_0^5 \frac{x^2}{e^x}\ dx \overset{\text{hint}}{=} \int_0^5 x^2 e^{-x}\ dx$

7. $\displaystyle\int (\ln x)^2\ dx$ hint: either $u = (\ln x)^2$ or $u = \ln x$. Which is more helpful? Double IBP either way.

8. $\displaystyle\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right)\ dx$

9. $\displaystyle\int x\arctan x\ dx$

10. $\displaystyle\int \ln\left(x^2+7\right)\ dx \overset{\text{hint}}{=} \int \left(\ln\left(x^2+7\right)\right)\cdot 1\ dx$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30-9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00–3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: 10–11:30 am

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

Do something different or new this week, for studying purposes.

Go to office hours and also go to Math Fellow TA hours