

Homework #5

Due Friday, September 19th in Gradescope by 11:59 pm

Goal: Exploring Limits using L'Hôpital's Rule, and solidifying size arguments**FIRST:** Read through and understand the following two Limit Examples.

$$\begin{aligned}
 \text{Ex : } \lim_{x \rightarrow 0} \frac{\arcsin x + \cos(3x) - e^x}{\arctan(3x) + x^2 - \sin(3x)} & \stackrel{\left(\frac{0}{0}\right)}{=} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}} - 3\sin(3x) - e^x}{\frac{1}{1+(3x)^2} \cdot (3) + 2x - 3\cos(3x)} \stackrel{\left(\frac{0}{0}\right)}{=} \\
 & \stackrel{\text{prep}}{=} \lim_{x \rightarrow 0} \frac{(1-x^2)^{-\frac{1}{2}} - 3\sin(3x) - e^x}{3(1+9x^2)^{-1} + 2x - 3\cos(3x)} \stackrel{\left(\frac{0}{0}\right)}{=} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{-\frac{1}{2}(1-x^2)^{-\frac{3}{2}} \cdot (-2x) - 9\cos(3x) - e^x}{-3(1+9x^2)^{-2} \cdot (18x) + 2 + 9\sin(3x)} \\
 & \stackrel{\text{rewrite}}{=} \lim_{x \rightarrow 0} \frac{\cancel{\frac{x}{(1-x^2)^{\frac{3}{2}}}} - \cancel{9\cos(3x)} - \cancel{e^x}}{\cancel{\frac{-54x}{(1+9x^2)^2}} + 2 + \cancel{9\sin(3x)}} = \frac{-9-1}{2} = \frac{-10}{2} = \boxed{-5}
 \end{aligned}$$

$$\text{Ex: } \lim_{x \rightarrow \infty} \left(1 - \frac{2}{x^3}\right)^{x^3} \stackrel{1^\infty}{=} \lim_{x \rightarrow \infty} e^{\ln\left(\left(1 - \frac{2}{x^3}\right)^{x^3}\right)} \stackrel{\infty \cdot 0}{=} \lim_{x \rightarrow \infty} x^3 \ln\left(1 - \frac{2}{x^3}\right)$$

$$\begin{aligned}
 & \lim_{x \rightarrow \infty} \frac{\ln\left(1 - \frac{2}{x^3}\right)}{\frac{1}{x^3}} \stackrel{\left(\frac{0}{0}\right)}{=} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{\left(\frac{1}{1 - \frac{2}{x^3}}\right) \left(\frac{6}{x^4}\right)}{-\frac{3}{x^4}} \\
 & = e
 \end{aligned}$$

$$\begin{aligned}
 & \lim_{x \rightarrow \infty} \left(\frac{1}{1 - \frac{2}{x^3}}\right) \left(\frac{6}{x^4}\right) \cdot \left(-\frac{x^4}{3}\right) = \lim_{x \rightarrow \infty} \left(\frac{1}{1 - \frac{2}{x^3}}\right) (-2) \\
 & = e
 \end{aligned}$$

Continue to NEXT Page for HW problems.

Compute each of the following Limits. Simplify. **You must mark each application of L'Hôpital's Rule. Justify every step.**

1. $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 - \sin \theta}{1 + \cos(2\theta)}$

2. $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$

3. $\lim_{x \rightarrow 0^+} \frac{\ln x}{x}$

4. $\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - 2x}{x^2}$

5. $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

6. $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

7. $\lim_{x \rightarrow 0} \frac{\arcsin(3x)}{\arctan(4x)}$

8. $\lim_{x \rightarrow 0} \frac{x - \arcsin x}{\arctan(2x) - 2x}$

9. $\lim_{x \rightarrow 0} \frac{3xe^x - \arctan(3x)}{x + \ln(1 - x)}$

10. $\lim_{x \rightarrow 0} \frac{\arcsin x + x^2 - x}{\cos x - \arctan(5x) - e^{-5x}}$

11. $\lim_{x \rightarrow \infty} x \sin\left(\frac{\pi}{x}\right)$

12. $\lim_{x \rightarrow \infty} x \ln\left(1 - \frac{1}{x}\right)$

13. $\lim_{x \rightarrow 0^+} x \ln x$

14. $\lim_{x \rightarrow 0^+} \sqrt{x} \ln x$

15. $\lim_{x \rightarrow \infty} x^2 e^{-x}$

16. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

17. $\lim_{x \rightarrow 0^+} (1 + \ln(1 - 3x))^{\frac{1}{x}}$

18. $\lim_{x \rightarrow \infty} \left(1 - \arctan\left(\frac{7}{x^4}\right)\right)^{x^4}$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30–9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00–3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: 10–11:30 am

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

Please e-mail with questions/concerns: dbenedetto@amherst.edu

Start early. Box your answers.

Show all details and justifications in a nice final draft. No Mess.