Math 121, Section(s) 01 and 02, Fall 2025

**Homework #4**

Due **Wednesday, September 17th** in Gradescope by 11:59 pm

**Goal:** Reviewing Inverse Trigonometric Functions and Limits (no L'Hopital's Rule yet)

**FIRST:** Read through and understand the following Examples.

Ex:

$$\int_e^{e^3} \frac{1}{x\left(3+(\ln x)^2\right)}\,dx = \int_1^3 \frac{1}{3+u^2}\,du \overset{\text{a-rule}}{=} \frac{1}{\sqrt{3}}\arctan\left(\frac{u}{\sqrt{3}}\right)\Bigg|_1^3$$

$$= \frac{1}{\sqrt{3}}\left(\arctan\left(\frac{3}{\sqrt{3}}\right)^{\sqrt{3}} - \arctan\left(\frac{1}{\sqrt{3}}\right)\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}\left(\frac{2\pi}{6}-\frac{\pi}{6}\right) = \boxed{\frac{\pi}{6\sqrt{3}}}$$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}} \qquad \boxed{\begin{aligned} x = e &\Rightarrow u = \ln e = 1 \\ x = e^3 &\Rightarrow u = \ln e^3 = 3 \end{aligned}}$$

Ex:

$$\int \frac{e^{3x}}{4+e^{3x}}\,dx = \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C = \boxed{\frac{1}{3}\ln\left|4+e^{3x}\right| + C}$$

$$\boxed{\begin{aligned} u &= 4+e^{3x} \\ du &= 3e^{3x}\,dx \\ \frac{1}{3}du &= e^{3x}\,dx \end{aligned}}$$

Ex:

$$\int \frac{e^{3x}}{4+e^{6x}}\,dx = \int \frac{e^{3x}}{4+(e^{3x})^2}\,dx \quad \text{look for the perfect square to ready for arctan rule}$$

$$\frac{1}{3}\int \frac{1}{4+u^2}\,du \overset{\text{a-rule}}{=} \frac{1}{3}\left(\frac{1}{2}\arctan\left(\frac{u}{2}\right)\right) + C = \boxed{\frac{1}{6}\arctan\left(\frac{e^{3x}}{2}\right) + C}$$

$$\boxed{\begin{aligned} u &= e^{3x} \\ du &= 3e^{3x}\,dx \\ \frac{1}{3}du &= e^{3x}\,dx \end{aligned}}$$

Next, Compute each of the following Integrals. Simplify.

1. $\displaystyle\int_2^{2\sqrt{3}} \frac{1}{\sqrt{16-x^2}}\,dx$
2. $\displaystyle\int_0^{\ln 3} \frac{e^x}{3+e^{2x}}\,dx$
3. $\displaystyle\int_0^{\ln\sqrt{3}} \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$
4. $\displaystyle\int_4^{4\sqrt{3}} \frac{1}{16+x^2}\,dx$
5. $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$
6. $\displaystyle\int \frac{x^2}{x^2+4}\,dx$
7. $\displaystyle\int \frac{2x^2+5}{x^2+1}\,dx$
8. $\displaystyle\int \frac{1}{(1+x^2)\,(5+(\arctan x)^2)}\,dx$
9. $\displaystyle\int_3^9 \frac{1}{\sqrt{x}\,(x+9)}\,dx$
10. $\displaystyle\int \frac{x^2+x+1}{x^2+4}\,dx$

Compute each of the following Limits. Simplify. Use arrows to justify the size arguments.

11. $\displaystyle\lim_{x\to 5^+} \frac{1}{x-5}$
12. $\displaystyle\lim_{x\to 5^-} \frac{1}{x-5}$
13. $\displaystyle\lim_{x\to 8^+} \ln|x-8|$
14. $\displaystyle\lim_{x\to 8^-} \ln|x-8|$
15. $\displaystyle\lim_{x\to 3^+} e^{\frac{2}{x-3}}$
16. $\displaystyle\lim_{x\to 3^-} e^{\frac{2}{x-3}}$
17. $\displaystyle\lim_{x\to\infty} \ln\left(1-\arctan\left(\frac{5}{x^4}\right)\right)$
18. $\displaystyle\lim_{x\to\infty} \ln\left(\frac{\pi}{2}-\arctan x\right)$
19. $\displaystyle\lim_{x\to 4^-} \ln|\ln|x-4||$
20. $\displaystyle\lim_{x\to 0^+} \arctan\left(\frac{\ln x}{5}\right)$

21. Present two different methods to Prove that $\displaystyle\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

# REGULAR OFFICE HOURS

## Monday: 12:00–3:00 pm

**6–9:00pm TAs Emma/Myles, SMUDD 204**

## Tuesday: 1:00–4:00 pm

**5:30–7:00 pm TA Julia, SMUDD 204**

**7:30-9:00 pm TA Emma, SMUDD 204**

## Wednesday: 1:00–3:00 pm

**6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204**

## Thursday: 10–11:30 am

**7:30–10:30 pm TAs Natalie/DJ, SMUDD 204**

## Friday: 12:00–2:00 pm

**7:30–9:00 pm TA DJ, SMUDD 204**

• Please stop by for help! Try to attend at least one office hour for me and at least one for the Math Fellows each week.

• You can also find help at the Math Fellow (Emma, Myles, Natalie, Julia, DJ) or the QCenter help sessions.