Math 121, Section(s) 01 and 02, Fall 2025

Homework #3

Due **Friday, September 12th** in Gradescope by 11:59 pm

Goal: Solidying Calculus for Inverse Sine and Inverse Tangent.

FIRST: Read through and understand the following two Derivative proofs.

Ex: **PROVE** that $\dfrac{d}{dx}\arctan x = \dfrac{1}{1+x^2}$

Proof: Let $y = \arctan x$ Looking to solve for $\dfrac{dy}{dx}$

Invert $\tan y = x$

Differentiate $\dfrac{d}{dx}(\tan y) = \dfrac{d}{dx}(x)$

$$\sec^2 y \cdot \frac{dy}{dx} = 1$$

Solve $$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+(\tan y)^2} = \frac{1}{1+x^2}$$

Ex: **PROVE** that $\dfrac{d}{dx}\arcsin x = \dfrac{1}{\sqrt{1-x^2}}$

Proof: Let $y = \arcsin x$ Looking to solve for $\dfrac{dy}{dx}$

Invert $\sin y = x$

Differentiate $\dfrac{d}{dx}(\sin y) = \dfrac{d}{dx}(x)$

$$\cos y \cdot \frac{dy}{dx} = 1$$

Solve $$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-(\sin y)^2}} = \frac{1}{\sqrt{1-x^2}}$$

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Differentiate the following functions for 1-4. Simplify.

1. $f(x) = \tan^{-1}(x^2)$
2. $f(x) = (\tan^{-1}(x))^2$
3. $y = x\sin^{-1}x + \sqrt{1-x^2}$
4. $f(x) = \ln\left(1 - \arcsin\left(\frac{2}{x^4}\right)\right)$
5. Find the value of the expression $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$
6. Simplify the expression $\sin(\tan^{-1}x)$
7. Compute the Second Derivative for $f(x) = \arctan(2x)$
8. Compute the Second Derivative for $f(x) = \arcsin(6x)$
9. **Prove** that $\frac{d}{dx}\sin^{-1}(3x) = \frac{3}{\sqrt{1-9x^2}}$
10. **Prove** that $\frac{d}{dx}\tan^{-1}(5x) = \frac{5}{1+25x^2}$
11. Use Integration to **Justify** that $\int \frac{1}{3+x^2}\,dx = \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$

Compute each of the following Integrals. Simplify.

12. $\int \frac{x^2}{x^2+1}\,dx$
13. $\int \frac{x+1}{x^2+1}\,dx$
14. $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2}\,dx$
15. $\int_0^{\frac{1}{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$
16. $\int \frac{1}{\sqrt{1-x^2}\cdot\sin^{-1}x}\,dx$
17. $\int_1^3 \frac{1}{\sqrt{x}\,(1+x)}\,dx$
18. $\int_0^{\ln 3} \frac{e^x}{1+e^x}\,dx$
19. $\int_0^{\frac{1}{2}\ln 3} \frac{e^x}{1+e^{2x}}\,dx$
20. $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\,dx$
21. $\int_3^{3\sqrt{3}} \frac{1}{\sqrt{36-x^2}} + \frac{1}{9+x^2}\,dx$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30-9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00–3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: 10–11:30 am

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

- Please do not wait until the last night to start.
- Please stop by for help! Please try the homework before you come by though. Final Answer keys are posted on the webpage. Please do **not** look at them unless you have completed the problems. **They are not a replacement for my help or your understanding.**
- You can also find help at the Math Fellow sessions or the QCenter hours with Tim St. Onge