

Homework #18

Due **Friday, November 21st** in Gradescope by 11:59 pm ET**Goal:** Exploring More complicated Sums and Limits using Series.**FIRST:** Read through and understand the following Examples.Find the **sum** of each of the following series (which do converge). Simplify.TIP: For these values, it could help to write out the general MacLaurin Series in x first.

$$\text{Ex: } -1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} + \dots = - \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots \right) = -\arctan 1 = \boxed{-\frac{\pi}{4}}$$

$$\begin{aligned} \text{Ex: } \sum_{n=0}^{\infty} \frac{(-1)^n (\ln 64)^n}{2^{n+1} n!} &= \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-1)^n (\ln 64)^n}{2^n n!} = \frac{1}{2} \sum_{n=0}^{\infty} \frac{\left(-\frac{\ln 64}{2}\right)^n}{n!} = \frac{1}{2} e^{-\frac{\ln 64}{2}} = \frac{1}{2} e^{-\frac{1}{2} \ln(64)} \\ &= \frac{1}{2} e^{\ln((64)^{-\frac{1}{2}})} = \frac{1}{2} \cdot (64)^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{(64)^{\frac{1}{2}}} = \frac{1}{2} \cdot \frac{1}{\sqrt{64}} = \frac{1}{2} \cdot \frac{1}{8} = \boxed{\frac{1}{16}} \end{aligned}$$

$$\begin{aligned} \text{Ex: } \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \pi^{2n}}{9^n (2n+1)!} &= - \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n} (2n+1)!} = - \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{3}\right)^{2n}}{(2n+1)!} \cdot \frac{\pi}{3} \\ &= -\frac{1}{\frac{\pi}{3}} \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{3}\right)^{2n+1}}{(2n+1)!} = -\frac{3}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{3}\right)^{2n+1}}{(2n+1)!} = -\frac{3}{\pi} \sin\left(\frac{\pi}{3}\right) = \boxed{-\frac{3\sqrt{3}}{2\pi}} \end{aligned}$$

$$\text{Ex: } 1 + 1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \frac{\pi^8}{8!} - \dots = 1^{\text{extra}} + \cos \pi = 1 + \cos \pi = 1 - 1 = \boxed{0}$$

$$\text{Ex: } -\frac{2}{2} + \frac{2}{3} - \frac{2}{4} + \frac{2}{5} - \dots = 2 \left(-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots \right) = 2 (\ln(1+1) - 1) = \boxed{2 \ln 2 - 2}$$

Ex: Use series to compute this **Limit**.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x^2 + x - \arctan x}{1 - 3x - e^{-3x}} &= \lim_{x \rightarrow 0} \frac{x^2 + x - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \right)}{1 - 3x - \left(1 - 3x + \frac{3^2 x^2}{2!} - \frac{3^3 x^3}{3!} + \dots \right)} \\ &= \lim_{x \rightarrow 0} \frac{x^2 + \frac{x^3}{3} - \frac{x^5}{5} + \dots}{-\frac{3^2 x^2}{2!} + \frac{3^3 x^3}{3!} - \frac{3^4 x^4}{4!} + \dots} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \rightarrow 0} \frac{1 + \frac{x}{3} - \frac{x^3}{5} + \dots}{-\frac{9}{2!} + \frac{3^3 x}{3!} - \frac{3^4 x^2}{4!} + \dots} = \frac{1}{-\frac{9}{2}} = \boxed{-\frac{2}{9}} \end{aligned}$$

Continue on and Complete the next page of problems

Find the **sum** of each of the following series (which do converge). Simplify.

1. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$

2. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{9^n (2n)!}$

3. $-\frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \frac{\pi^9}{9!} - \dots$

4. $-\frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$

5. $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \dots$

6. $\sum_{n=0}^{\infty} \frac{(-1)^n (\ln 8)^n}{3^{n+1} n!}$

7. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(36)^n (2n+1)!}$

8. $\frac{1}{6} - \frac{1}{2(6)^2} + \frac{1}{3(6)^3} - \frac{1}{4(6)^4} + \dots$

9. $1 - e + \frac{e^2}{2!} - \frac{e^3}{3!} + \frac{e^4}{4!} - \frac{e^5}{5!} + \dots$

10. $-\frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \frac{\pi^8}{8!} - \dots$

11. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!}$

12. $\sum_{n=0}^{\infty} \frac{1}{e^n} \stackrel{\text{hint}}{=} \sum_{n=0}^{\infty} \left(\frac{1}{e}\right)^n$

13. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^{n+1} (\ln 9)^n}{n!}$

14. $4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \dots$

15. $\sum_{n=0}^{\infty} \frac{e^6 (x-6)^n}{n!}$ (answer will be in x)

16. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \pi^{2n+1}}{9 (2n)!}$

17. $\sum_{n=0}^{\infty} \frac{1}{3! \pi^n}$

18. $-\pi + \frac{\pi^3}{3!} - \frac{\pi^5}{5!} + \dots$

19. $1 + 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

20. $2 - 1 + \frac{2}{3} - \frac{2}{4} + \frac{2}{5} - \dots$

21. $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots$

22. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(\sqrt{2})^{4n} (2n)!}$

23. Use Series to Compute $\lim_{x \rightarrow 0} \frac{xe^x - \arctan x}{\ln(1+5x) - 5x}$ Check your answer using L'H Rule

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30–9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00–3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: 10–11:30 am

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

Match the sum formulas precisely.

Pattern find and check your guess too.

Happy short vacation!