Math 121, Section(s) 01, 02, Fall 2025

Homework #14

Due **Wednesday, October 29th** in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Absolute and Conditional Convergence...also using the Absolute Convergence Test. Finally... some review problems.

FIRST: Read through and understand the following Examples. Determine whether the Series is Absolutely Convergent, Conditionally Convergent, or Divergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2+7}{n^7+2} \hookrightarrow \text{A.S.} \sum_{n=1}^{\infty} \frac{n^2+7}{n^7+2} \approx \sum_{n=1}^{\infty} \frac{n^2}{n^7} = \sum_{n=1}^{\infty} \frac{1}{n^5} \text{ Converges } p\text{-series } p=5>1.$$

$$\text{Check: } \lim_{n\to\infty} \frac{\frac{n^2+7}{n^7+2}}{\frac{1}{n^5}} = \lim_{n\to\infty} \frac{n^7+7n^5}{n^7+2} \cdot \frac{\frac{1}{n^7}}{\frac{1}{n^7}} = \lim_{n\to\infty} \frac{1+\frac{7}{n^2}\to 0}{1+\frac{2}{n^7}\to 0} = 1 \text{ Finite/Non-zero}$$

The **Absolute Series** $\sum_{n=1}^{\infty} \frac{n^2+7}{n^7+2}$ also **Converges** by Limit Comparison Test (LCT).

Finally, the Original Series is $\boxed{\text{Absolutely Convergent (A.C.)}}$ (by Definition).

$$\text{Ex: } \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{7n+3} \hookrightarrow \text{A.S.} \sum_{n=1}^{\infty} \frac{1}{7n+3} \approx \sum_{n=1}^{\infty} \frac{1}{n} \text{ Divergent Harmonic } p\text{-Series } p=1$$

$$\text{Check: } \lim_{n\to\infty} \frac{\frac{1}{7n+3}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{7n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{7+\frac{3}{n}\to 0} = \frac{1}{7} \text{ Finite/Non-zero.}$$

Therefore, the **Absolute Series** also **Diverges** by Limit Comparison Test.

Now, we must examine the original alternating series with the Alternating Series Test.

- Isolate $b_n = \frac{1}{7n+3} > 0$
- $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{7n+3} \to \frac{0}{\infty} = 0$
- Terms Decreasing $b_{n+1} < b_n$ because $b_{n+1} = \frac{1}{7(n+1)+3} = \frac{1}{7n+10} < \frac{1}{7n+3} = b_n$

Therefore, the **Original Series Converges** by the Alternating Series Test. Finally, we can conclude the Original Series is $\boxed{\text{Conditionally Convergent (C.C.)}}$ (by Definition).

Now complete the following HW problems

Determine whether the given series is Absolutely Convergent, Conditionally Convergent or Divergent. Number 3 is Ratio Test, but use the AC and CC *charts* for 1, 2, 4, 5.

Make sure to look at the sample problems (above) first for guidance.

1. $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n^3+8}{n^8+3}$ 2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+1}$ 3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (\ln n) (2n)!}{n^n \, 2^{3n} \, (n!)}$

4. $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n+1}{n^2}$ 5. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n^3}{n^7+2}$

6. Write the statement of the Absolute Convergence Test.

7. Use the Absolute Convergent Test to show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$ Converges.

8. Use the Absolute Convergent Test to show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin^2 n}{n^8+2}$ Converges.

Review

9. Show that the Sequence $\displaystyle\left\{\left(\frac{n}{n+1}\right)^n\right\}_{n=1}^{\infty}$ Converges to $\dfrac{1}{e}$. Hint: See HW 6, #2.

10. Determine if the Series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$ Converges or Diverges.

Hint: Reference the work/conclusion from 9 above.

11. Find the Sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \, 5^{2n+1}}{2^{5n-1}}$.

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30-9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00-3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: 10–11:30 am

extras may be added, TBD weekly

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

This is the end of the material for the Exam 2. Material stops after Section 11.6 Absolute Convergence Test and Ratio Test and Absolute/Conditional Convergence.