Math 121, Section(s) 01, 02, Fall 2025

Homework #11

Due **Friday, October 17th** in Gradescope by 11:59 pm ET

Goal: Exploring Convergence of Infinite Series. Focus on Geometric Series and the n^{th} Term Divergence Test. We may also need L'Hôpital's Rule to finish some of the limits at hand.

FIRST: Read through and understand the following Examples. Determine whether the given Series Converges or Diverges. If it Converges, find the Sum value. Justify.

Ex: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 5^{n-1}}{3^{2n+1}} = -\frac{1}{3^3} + \frac{5}{3^5} - \frac{5^2}{3^7} + \frac{5^3}{3^9} + \dots$ Here $a = -\dfrac{1}{27}$ and $r = -\dfrac{5}{3^2} = -\dfrac{5}{9}$.

Series **Converges by Geometric Series Test (GST)**, because $|r| = \left|-\dfrac{5}{9}\right| = \dfrac{5}{9} < 1$ with

$$\text{SUM}=\frac{a}{1-r} = \frac{-\frac{1}{27}}{1-\left(-\frac{5}{9}\right)} = \frac{-\frac{1}{27}}{\frac{14}{9}} = -\frac{1}{\cancel{27}_{\,3}} \cdot \frac{\cancel{9}}{14} = -\frac{1}{3}\cdot\frac{1}{14} = \boxed{-\frac{1}{42}}$$

Ex: $\displaystyle\sum_{n=0}^{\infty} \left(\frac{7}{3}\right)^n = 1 + \frac{7}{3} + \frac{7^2}{3^2} + \frac{7^3}{3^3} + \dots$ Here $a = 1$ and $r = \dfrac{7}{3}$.

Series **Diverges by GST**, because $|r| = \dfrac{7}{3} \geq 1$.

Ex: $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ **Diverges by the n^{th} Term Divergence Test (nTDT)** because

$$\lim_{n\to\infty} \frac{e^n}{n^2}\,\frac{\infty}{\infty} = \lim_{x\to\infty} \frac{e^x}{x^2}\,\frac{\infty}{\infty} \overset{\text{L'H}}{=} \lim_{x\to\infty} \frac{e^x}{2x}\,\frac{\infty}{\infty} \overset{\text{L'H}}{=} \lim_{x\to\infty} \frac{\overset{\nearrow\infty}{e^x}}{2} = \infty \neq 0$$

Ex: $\displaystyle\sum_{n=1}^{\infty} 3$ **Diverges by nTDT** because $\displaystyle\lim_{n\to\infty} 3 = 3 \neq 0$ Q: Is this also Geometric? $r =$?

Ex: $\displaystyle\sum_{n=1}^{\infty} e^{\frac{1}{n}}$ **Diverges by nTDT** because $\displaystyle\lim_{n\to\infty} e^{\frac{1}{n}} = 1 \neq 0$

Continue to NEXT Page for HW problems.

Determine whether each of the following Converge or Diverge. Justify.

1. $\{8\}_{n=1}^{\infty}$

2. $\sum_{n=1}^{\infty} 8$

3. $\left\{\dfrac{2n}{3n+1}\right\}_{n=1}^{\infty}$

4. $\sum_{n=1}^{\infty} \dfrac{2n}{3n+1}$

Determine whether the given series Converges or Diverges. If it converges, find the Sum value. Justify.

5. $\sum_{n=1}^{\infty} \dfrac{8}{5^n}$

6. $\sum_{n=0}^{\infty} \dfrac{8}{5^n}$

7. $\sum_{n=1}^{\infty} \dfrac{4^n}{9^{n-1}}$

8. $\sum_{n=1}^{\infty} \dfrac{7^{n+1}}{3^n}$

9. $\sum_{n=1}^{\infty} (-1)^n \dfrac{4^{2n+1}}{3^{3n-1}}$

10. $\sum_{n=1}^{\infty} e^n$

11. $\sum_{n=1}^{\infty} \dfrac{1+2^n}{3^n}$

12. $\sum_{n=0}^{\infty} \dfrac{1}{(1999)^n}$

13. $\sum_{n=1}^{\infty} \dfrac{1}{1999}$

14. $\sum_{n=1}^{\infty} \arctan n$

15. $\sum_{n=2}^{\infty} \dfrac{n^2}{\ln n}$

16. $\sum_{n=1}^{\infty} \sin^2\left(\dfrac{\pi n^4+1}{3n^4+5}\right)$

17. $\sum_{n=1}^{\infty} \left(1+\ln\left(1+\dfrac{5}{n}\right)\right)^n$

Consider these variable versions of Geometric Series. Find the values of x for which the series Converges. Find the sum of the Series for those values of x (answer in terms of x).

18. $\sum_{n=1}^{\infty} (-5)^n x^n$

19. $\sum_{n=0}^{\infty} \dfrac{(x-2)^n}{3^n}$

HINT for 18/19: Recall that for Absolute Values in Inequalities

$$|A| < B \text{ translates as } -B < A < B.$$

For example: $|x-7| < 1$ translates as $-1 < x-7 < 1$ or $6 < x < 8$.

KEY: You must handle the Absolute value Inequality translation before working the algebra.

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30-9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00-3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: 10–11:30 am

extras may be added, TBD weekly

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

Challenge yourself to work differently this week...

Catch an office hour a day?! Either daytime or evening time