

Homework #10

Due **Friday, October 10th** in Gradescope by 11:59 pm ET

Goal: Exploring Limits of Infinite Sequences. We may also need L'Hôpital's Rule to finish some of the limits at hand.

FIRST: Read through and understand the following 5 Examples.

Determine whether the given sequence Converges or Diverges. If it converges, find the Limit.

$$\text{Ex: } \left\{ \frac{\ln n}{n^3} \right\}_{n=1}^{\infty} \hookrightarrow \lim_{n \rightarrow \infty} \frac{\ln n}{n^3} \stackrel{\infty}{=} \lim_{x \rightarrow \infty} \frac{\ln x}{x^3} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{3x^2} = \lim_{x \rightarrow \infty} \frac{1}{3x^3} \xrightarrow{0} \boxed{0} \text{ Converges}$$

$$\text{Ex: } \left\{ \frac{e^n}{n^2} \right\}_{n=1}^{\infty} \hookrightarrow \lim_{n \rightarrow \infty} \frac{e^n}{n^2} \stackrel{\infty}{=} \lim_{x \rightarrow \infty} \frac{e^x}{x^2} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{e^x}{2x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{e^x}{2} \xrightarrow{\infty} \boxed{\infty} \text{ Diverges}$$

$$\text{Ex: } \left\{ \frac{4 - 9n^3}{5n^3 + 8n^2 - 7n - 6} \right\}_{n=1}^{\infty} \text{ Note: Can switch to the Related Function in } x \text{ and use L'H Rule or ... Quicker Algebra ...}$$

$$\hookrightarrow \lim_{n \rightarrow \infty} \frac{4 - 9n^3}{5n^3 + 8n^2 - 7n - 6} \frac{\left(\frac{1}{n^3} \right)}{\left(\frac{1}{n^3} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n^3} - 9}{5 + \frac{8}{n} - \frac{7}{n^2} - \frac{6}{n^3}} \xrightarrow{0} \boxed{-\frac{9}{5}} \text{ Converges}$$

$$\text{Ex: } \left\{ \left(1 - \sin \left(\frac{6}{n^3} \right) \right)^{n^3} \right\}_{n=1}^{\infty} \hookrightarrow \lim_{n \rightarrow \infty} \left(1 - \sin \left(\frac{6}{n^3} \right) \right)^{n^3} \stackrel{1^{\infty}}{=} \lim_{x \rightarrow \infty} \left(1 - \sin \left(\frac{6}{x^3} \right) \right)^{x^3}$$

$$= e^{\lim_{x \rightarrow \infty} \ln \left[\left(1 - \sin \left(\frac{6}{x^3} \right) \right)^{x^3} \right]} = e^{\lim_{x \rightarrow \infty} x^3 \ln \left(1 - \sin \left(\frac{6}{x^3} \right) \right)} \stackrel{\infty \cdot 0}{=} e^{\lim_{x \rightarrow \infty} \frac{\ln \left(1 - \sin \left(\frac{6}{x^3} \right) \right)}{\frac{1}{x^3}}}$$

$$\stackrel{\text{L'H}}{=} e^{\lim_{x \rightarrow \infty} \frac{\frac{1}{1 - \sin \left(\frac{6}{x^3} \right)} \cdot \left(-\cos \left(\frac{6}{x^3} \right) \right)^{-1} \cdot \left(-\frac{18}{x^4} \right)^6}{-\frac{3}{x^4}}} = e^{-6} = \boxed{\frac{1}{e^6}} \text{ Converges}$$

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Ex:

$$\left\{ \frac{(3n-1)!}{(3n+1)!} \right\}_{n=1}^{\infty}$$

$$\hookrightarrow \lim_{n \rightarrow \infty} \frac{(3n-1)!}{(3n+1)!} = \lim_{n \rightarrow \infty} \frac{(3n-1)!}{(3n+1)(3n)(3n-1)!} = \lim_{n \rightarrow \infty} \frac{1}{(3n+1)(3n)} \xrightarrow{0} 0 \quad \boxed{0} \text{ Converges}$$

Next do the following HW problems.

List the first five terms of the Sequence. (Start with $n = 1$)

$$1. \ a_n = \frac{(-1)^{n-1}}{5^n}$$

$$2. \ a_n = \frac{1}{(n+1)!}$$

$$3. \ a_n = \frac{(-1)^n n^2}{n+1}$$

Determine whether the given sequence Converges or Diverges. If it converges, find the Limit. Justify, no guessing here.

$$4. \ \left\{ \frac{n}{n+1} \right\}_{n=1}^{\infty}$$

$$5. \ \left\{ \frac{5n^2 + 3}{2n^2 - 7n} \right\}_{n=1}^{\infty}$$

$$6. \ \left\{ \frac{3n^4 - n - 5}{7n^4 + n^2 - 9} \right\}_{n=1}^{\infty}$$

$$7. \ \left\{ \frac{\tan^{-1} n}{n} \right\}$$

$$8. \ \left\{ \frac{n^2}{e^n} \right\}$$

$$9. \ \left\{ n \sin \left(\frac{1}{n} \right) \right\}$$

$$10. \ \left\{ \frac{(\ln n)^2}{n} \right\}_{n=1}^{\infty}$$

$$11. \ \left\{ \frac{n^{99}}{\ln n} \right\}_{n=2}^{\infty}$$

$$12. \ \left\{ \frac{\ln(99)}{n^{99}} \right\}$$

$$13. \ \left\{ \left(1 + \frac{1}{n} \right)^n \right\}_{n=1}^{\infty}$$

$$14. \ \left\{ \left(1 - \frac{5}{n^6} \right)^{n^6} \right\}_{n=1}^{\infty}$$

$$15. \ \left\{ \left(1 - \arcsin \left(\frac{3}{n^2} \right) \right)^{n^2} \right\}$$

$$16. \ \{ \ln(2n^2 + 1) - \ln(n^2 + 1) \}$$

$$17. \ \left\{ \frac{(n+3)!}{(n+1)!} \right\}_{n=1}^{\infty}$$

$$18. \ \left\{ \frac{(2n-1)!}{(2n+1)!} \right\}$$

$$19. \ \left\{ \cos^2 \left(\frac{\pi n^6 + 6}{6n^6 + 1} \right) \right\}_{n=1}^{\infty}$$

$$20. \ \left\{ \arctan \left(\frac{5n^7 + 1}{5n^7 + 7} \right) \right\}_{n=1}^{\infty}$$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

6–9:00pm TAs Emma/Myles, SMUDD 204

Tuesday: 1:00–4:00 pm

5:30–7:00 pm TA Julia, SMUDD 204

7:30-9:00 pm TA Emma, SMUDD 204

Wednesday: 1:00-3:00 pm

6–10:30 pm TAs Julia/Myles/Natalie, SMUDD 204

Thursday: none for Professor

extras may be added, TBD weekly

7:30–10:30 pm TAs Natalie/DJ, SMUDD 204

Friday: 12:00–2:00 pm

7:30–9:00 pm TA DJ, SMUDD 204

dig deep, check notation, reference, justify, search, clarify...

challenge to everyone this week, get help on a challenging problem
in office hours with me or a Math Fellow