Math 121 Midterm Exam #2 October 31, 2025

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\arctan(\sqrt{3})$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [30 Points] Compute the following **Improper** integrals. Simplify all answers. Justify.

(a) $\displaystyle\int_0^e x^4 \ln x \, dx$ (b) $\displaystyle\int_{-\infty}^1 \frac{1}{x^2-6x+13} \, dx$ (c) $\displaystyle\int_{-7}^0 \frac{x+15}{x^2+6x-7} \, dx$

2. [12 Points]

Use the **Absolute Convergence Test** to show $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^4}$ **Converges**.

IMPORTANT: You are required to use the **Integral Test on the Absolute Series** in this problem.

Note: You do **not** have to check the 3 pre-conditions for the Integral Test.

3. [8 Points] **CREATE** a Series that **Diverges by the n^{th} Term Divergence Test AND** requires L'Hôpital's Rule in the Divergence Test's Limit.

Continue on to prove that the Series Diverges by the n^{th} Term Divergence Test.

4. [20 Points] Determine whether each of the given series **Converges** or **Diverges**. Name any Convergence Test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty}(2025)!$ (b) $-4-\frac{4}{2}-\frac{4}{3}-1-\frac{4}{5}-\frac{4}{6}-\dots$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^6}+\frac{(-6)^n}{7^{2n}}$

5. [30 Points] Determine whether each of the given series is **Absolutely Convergent**, **Conditionally Convergent**, or **Divergent**. Name any Convergence Test(s) you use, and justify all of your work.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^4+6}{n^6+4}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^n \cdot (2n)!}{6^n (n!)^3}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{6n+4}$

****

OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

****

OPTIONAL BONUS #1 We have seen that the harmonic series is a **Divergent series whose terms do indeed approach zero** as $n \to \infty$. Show that the following series $\displaystyle\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$ is another series with this property.