Math 121, Section(s) 01, 02, Fall 2024

Homework #9

Due **Wednesday, October 9th** in Gradescope by 11:59 pm ET

Goal: Exploring Improper Integrals, for both Type I (unbounded domain) and Type II (unbounded range). We will need IBP, Complete the Square, Partial Fractions, and some u-sub here. We may also need L'Hôpital's Rule to finish a few of the limits at hand.

FIRST: Read through and understand the following three Examples.

Ex:

$$\int_{-\infty}^{7} \frac{1}{x^2-6x+25}\,dx = \lim_{t\to-\infty}\int_t^7 \frac{1}{x^2-6x+25}\,dx \overset{\substack{\text{complete}\\ \text{square}}}{=} \lim_{t\to-\infty}\int_t^7 \frac{1}{(x-3)^2+16}\,dx$$

$$= \lim_{t\to-\infty}\int_{t-3}^{4}\frac{1}{u^2+16}\,du = \lim_{t\to-\infty}\frac{1}{4}\arctan\left(\frac{u}{4}\right)\Bigg|_{t-3}^{4} = \lim_{t\to-\infty}\frac{1}{4}\left(\arctan\left(\frac{4}{4}\right)-\arctan\left(\frac{t-3}{4}\right)\right)$$

$$= \lim_{t\to-\infty}\frac{1}{4}\left(\arctan(1)-\arctan\left(\cancelto{-\infty}{\frac{t-3}{4}}\right)\right) = \frac{1}{4}\left(\frac{\pi}{4}-\left(-\frac{\pi}{2}\right)\right) = \frac{1}{4}\left(\frac{3\pi}{4}\right) = \boxed{\frac{3\pi}{16}}$$

$$\boxed{\begin{aligned} u &= x-3 \\ du &= dx \end{aligned}} \qquad \boxed{\begin{aligned} &x=t \Rightarrow u=t-3 \\ &x=7 \Rightarrow y=7-3=4 \end{aligned}}$$

Ex:

$$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{s\to0^+}\int_s^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{s\to0^+}\int_s^1 (\ln x)x^{-\frac{1}{2}}\,dx = \lim_{s\to0^+} 2\sqrt{x}\ln x\Bigg|_s^1 - 2\int_s^1 \frac{1}{\sqrt{x}}\,dx$$

$$= \lim_{s\to0^+} 2\sqrt{x}\ln x\Bigg|_s^1 - 4\sqrt{x}\Bigg|_s^1 = \lim_{s\to0^+} 2\cancelto{0}{\ln 1} - 4\cancelto{1}{\sqrt{1}} - \left(2\sqrt{s}\ln s^{0\cdot(-\infty)} - 4\cancelto{0}{\sqrt{s}}\right)$$

$$\overset{*}{=} \boxed{-4}$$

$$\text{IBP}\quad \boxed{\begin{aligned} u &= \ln x & dv &= x^{-\frac{1}{2}}dx \\ du &= \frac{1}{x}\,dx & v &= 2\sqrt{x} \end{aligned}}$$

$$\text{* L'H Finish: } \lim_{s\to0^+}\sqrt{s}\ln s \overset{0\cdot(-\infty)}{=} \lim_{s\to0^+}\frac{\ln s}{\frac{1}{\sqrt{s}}} \overset{\text{L'H}}{\underset{}{=}}{}^{\frac{-\infty}{\infty}} \lim_{s\to0^+}\frac{\frac{1}{s}}{-\frac{1}{2s^{\frac{3}{2}}}} = \lim_{s\to0^+} -2\sqrt{s} = 0$$

Continue to NEXT Page

Ex:

$$\int_0^6 \frac{8}{x^2-4x-12}\,dx = \int_0^6 \frac{8}{(x-6)(x+2)}\,dx = \lim_{t\to 6^-}\int_0^t \frac{8}{(x-6)(x+2)}\,dx$$

$$\overset{\text{PFD}}{=} \lim_{t\to 6^-}\int_0^t \frac{1}{x-6} - \frac{1}{x+2}\,dx = \lim_{t\to 6^-} \ln|x-6| - \ln|x+2|\Big|_0^t$$

$$= \lim_{t\to 6^-} \overset{-\infty}{\ln|\overset{0^+}{t-6}|} - \ln|t+2| - (\ln|-6| - \ln 2)$$

$$= -\infty - \ln 8 - \ln 6 + \ln 2 = \boxed{-\infty} \text{ Diverges}$$

PFD:

$$\frac{8}{(x-6)(x+2)} = \frac{A}{x-6} + \frac{B}{x+2}$$

Clearing the denominator yields:

$8 = A(x+2) + B(x-6)$
$8 = Ax + 2A + Bx - 6B$
$8 = (A+B)x + (2A-6B)$
so that $A+B=0$ and $2A-6B=8$
Solve for $A=1$ and $B=-1$

Compute each of the following Integrals. Simplify when possible.

1. $\displaystyle\int_{-\infty}^0 \frac{1}{3-4x}\,dx$

2. $\displaystyle\int_1^\infty \frac{1}{(2x+1)^3}\,dx$

3. $\displaystyle\int_2^\infty \frac{x}{e^{3x}}\,dx$

4. $\displaystyle\int_e^\infty \frac{\ln x}{x^3}\,dx$

5. $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$

6. $\displaystyle\int_e^\infty \frac{1}{x\ln x}\,dx$

7. $\displaystyle\int_{-\infty}^7 \frac{1}{x^2-4x+29}\,dx$

8. $\displaystyle\int_0^5 \frac{6}{x^2-4x-5}\,dx$

9. $\displaystyle\int_0^{e^5} \frac{1}{x\left[25+(\ln x)^2\right]}dx$

10. $\displaystyle\int_1^2 \frac{1}{x\ln x}\,dx$

11. $\displaystyle\int_0^1 x\ln x\,dx$

REGULAR OFFICE HOURS

Sunday 6:00–9:00 pm TAs Natalie/Oscar, SMUDD 207

Monday: 12:00–3:00 pm

6:00–9:00 pm TAs Aaron/Oscar, SMUDD 207

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Gretta, SMUDD 207

Wednesday: 1:00-3:00 pm

7:30–9:00 pm TA Natalie, SMUDD 207

Thursday: none for Professor

extras may be added, TBD weekly

6–9:00 pm TAs Gretta/DJ, SMUDD 207

Friday: 12:00–3:00 pm

6:00–9:00 pm TAs Aaron/DJ, SMUDD 207

You are welcome at Office Hours all the time. *Please come!*