



Math 121 Exam 3 December 4, 2024



- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{3\ln 3}$, or $\arctan \sqrt{3}$ should be simplified.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [20 Points] Interval and Radius of Convergence. Analyze carefully, with full justification.

(a) Find the **Interval** and **Radius** of Convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n (3x+5)^n}{(3n+5)^2 \cdot 4^n}$

(b) Design a Power Series which is convergent only at $x = 6$. Once you create your series, then proceed to justify that the Interval of Convergence is indeed $I = \{6\}$.

2. [22 Points] Find the MacLaurin Series for each of the functions. Also **STATE** the Radius of Convergence for each series. Answers should be in sigma notation $\sum_{n=0}^{\infty}$. Simplify.

(a) $\ln(1+9x^2)$ (b) $x^3 e^{-4x}$ (c) $\frac{d}{dx}(8x^4 \sin(8x))$ (d) $\int \frac{x^2}{8+x^3} dx$

3. [10 Points] Use Series to Estimate $\frac{1}{\sqrt{e}} = e^{-\frac{1}{2}}$ with error less than $\frac{1}{100}$.

Simplify. Tip: $(16) \cdot (24) = 384$

4. [26 Points] Find the **sum** for each of the following convergent series. Simplify, if possible.

$$\begin{aligned}
 \text{(a)} \quad & \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n} (2n+1)!} & \text{(b)} \quad & -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \dots & \text{(c)} \quad & \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^n (2n)!} \\
 \text{(d)} \quad & \sum_{n=0}^{\infty} \frac{(-1)^{n+1} (\ln 9)^n}{2^n \cdot n!} & \text{(e)} \quad & 4 + 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \dots & \text{(f)} \quad & -\frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \frac{\pi^8}{8!} - \dots
 \end{aligned}$$

5. [10 Points] Use Series to compute the following Limit $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{e^{-x} - 1 + x}$

6. [12 Points] Find the MacLaurin Series Representation for $\arctan(x^4)$.

You **MUST** use Integration of Series, and the following Hint formula.

Hint: $\boxed{\arctan(x^4) = \int \frac{4x^3}{1+x^8} dx = \int 4x^3 \left(\frac{1}{1+x^8} \right) dx}$ P.S. Yes, solve for C

OPTIONAL: Feel free to check your answer using a substitution.