• Please see the course webpage for the answer key.

Compute each of the following Integrals.

1. $\displaystyle\int_3^{3\sqrt{3}} \frac{1}{\sqrt{36-x^2}} + \frac{1}{9+x^2}\ dx.$

2. $\displaystyle\int \frac{4}{(1+x^2)(1+(\arctan x)^2)}\ dx.$

3. $\displaystyle\int_0^{\ln\sqrt{2}} \frac{e^x}{\sqrt{4-e^{2x}}}\ dx.$

4. $\displaystyle\int_e^{e^3} \frac{1}{x\left(3+(\ln x)^2\right)}\ dx$

5. $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{\cos x}{3+\sin^2 x}\ dx$

6. $\displaystyle\int_3^9 \frac{1}{\sqrt{x}(9+x)}\ dx$

7. $\displaystyle\int \frac{x^2+x+1}{x^2+4}\ dx$

8. (a) Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\arctan x = \dfrac{1}{1+x^2}$

(b) From part (a) we now know that $\displaystyle\int \frac{1}{1+x^2}\ dx = \arctan x + C.$

Use this fact **and integration** to **PROVE** that $\displaystyle\int \frac{1}{3+x^2}\ dx = \frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right)+C$

9. Use implicit differentiation to **PROVE** that $\dfrac{d}{dx}\sin^{-1}(5x) = \dfrac{5}{\sqrt{1-25x^2}}$