Take Home Quiz 4 Fall2023 Answers

1. $\left\{\dfrac{3n^5-6n^2+7}{8n^5+2n^4-1}\right\}_{n=1}^{\infty}$ $\lim_{n\to\infty} \dfrac{3n^5-6n^2+7}{8n^5+2n^4-1} \cdot \dfrac{\frac{1}{n^5}}{\frac{1}{n^5}} = \lim_{n\to\infty} \dfrac{3-\frac{6}{n^3}+\frac{7}{n^5}}{8+\frac{2}{n}-\frac{1}{n^5}} = \dfrac{3}{8}$ Converges

2. $\left\{\dfrac{n^7}{4\ln n}\right\}_{n=1}^{\infty}$ $\lim_{n\to\infty}\dfrac{n^7}{4\ln n} \; \frac{\infty}{\infty} = \lim_{x\to\infty}\dfrac{x^7}{4\ln x} \; \frac{\infty}{\infty} \stackrel{L'H}{=} \lim_{x\to\infty}\dfrac{7x^6}{4\cdot\frac{1}{x}} \cdot \dfrac{x}{x} = \lim_{x\to\infty}\dfrac{7x^7}{4} = \infty$

Diverges

3. $\left\{\dfrac{(2n-1)!}{(2n+1)!}\right\}_{n=1}^{\infty}$ $\lim_{n\to\infty}\dfrac{(2n-1)!}{(2n+1)!} = \lim_{n\to\infty}\dfrac{\cancel{(2n-1)!}}{(2n+1)(2n)\cancel{(2n-1)!}} = \lim_{n\to\infty}\dfrac{1}{(2n+1)(2n)} = 0$

4. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n 3^{n+1}}{2^{3n-1}} = -\frac{3^2}{2^2} + \frac{3^3}{2^5} - \frac{3^4}{2^8} + \cdots$ ($n=1$, $n=2$, $n=3$)

$a = \dfrac{-3^2}{2^2} = \dfrac{-9}{4}$ Converges by GST b/c $|r| = \left|-\dfrac{3}{8}\right| = \dfrac{3}{8} < 1$

$r = \dfrac{-3}{2^3} = \dfrac{-3}{8}$ with Sum $= \dfrac{a}{1-r} = \dfrac{-\frac{9}{4}}{1-\left(-\frac{3}{8}\right)} = \dfrac{-\frac{9}{4}}{\frac{11}{8}} = \dfrac{-18}{11}$ Match!

check minus signs

5. $\displaystyle\sum_{n=2}^{\infty}\frac{n^7}{4\ln n}$ Diverges by nTDT since

$\lim_{n\to\infty}\dfrac{n^7}{4\ln n} \; \frac{\infty}{\infty} = \lim_{x\to\infty}\dfrac{x^7}{4\ln x} \; \frac{\infty}{\infty} \stackrel{L'H}{=} \lim_{x\to\infty}\dfrac{7x^6}{4\cdot\frac{1}{x}} \cdot \dfrac{x}{x} = \lim_{x\to\infty}\dfrac{7x^7}{4} = \infty \neq 0$

or see work in #2. above

6. $\displaystyle\sum_{n=1}^{\infty}\frac{4}{n^7}+\frac{4^n}{7^n} \stackrel{\text{split}}{=} \sum_{n=1}^{\infty}\frac{4}{n^7} + \sum_{n=1}^{\infty}\frac{4^n}{7^n}$

$\displaystyle = 4\sum_{n=1}^{\infty}\frac{1}{n^7} + \sum_{n=1}^{\infty}\left(\frac{4}{7}\right)^n$

Constant Multiple of Convergent p-Series $p=4>1$ is Convergent + Convergent by GST with $|r| = \left|\frac{4}{7}\right| = \frac{4}{7} < 1$

Original Series Converges b/c the Sum of 2 Convergent Series is Convergent.

(or by Arithmetic of Series)

7. $\sum_{n=1}^{\infty} 2023 = 2023 \overset{\times 1}{+} 2023 + 2023 + \cdots$

1 Diverges by GST because $|r| = |1| = 1 \geq 1$

OR

2. Diverges by nTDT because $\lim_{n \to \infty} 2023 = 2023 \neq 0$