Take Home Quiz 2 Fall 2023

1. $$\lim_{x\to 0} \frac{\ln(1-5x) + \arcsin(5x)}{3xe^{x} - \arctan(3x)} \qquad \frac{0}{0}$$

prep $-5(1-5x)^{-1}$ prep $5(1-25x^2)^{-1/2}$

$-5+5=0$
$0+3-3=0$

$$\underset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{1-5x}(-5) + \frac{1}{\sqrt{1-(5x)^2}}\cdot 5}{3xe^{x} + 3e^{x} - \frac{1}{1+(3x)^2}\cdot 3}$$

prep $-3(1+9x^2)^{-1}$

$$\underset{L'H}{=} \lim_{x\to 0} \frac{5(1-5x)^{-2}(-5) - \frac{5}{2}(1-25x^2)^{-3/2}(-50x)}{3xe^{x} + 3e^{x} + 3e^{x} + 3(1+9x^2)^{-2}(18x)}$$

rewrite

$$= \lim_{x\to 0} \frac{\frac{-25}{(1-5x)^2} + \cancel{\frac{250x}{2(1-25x^2)^{3/2}}}}{\cancel{3xe^{x}} + 6e^{x} + \cancel{\frac{54x}{(1+9x^2)^2}}} = \boxed{-\frac{25}{6}}$$

-25, $125x$, $\to 0$, 0, 6, $\to 0$

Match!

2. $$\lim_{x\to\infty}\left(1-\frac{8}{x^3}\right)^{x^3} = e^{\lim_{x\to\infty} \ln\left[\left(1-\frac{8}{x^3}\right)^{x^3}\right]} \qquad 1^{\infty}$$

$\infty \cdot 0$

$$= e^{\lim_{x\to\infty} x^3 \ln\left(1-\frac{8}{x^3}\right)}$$

FLIP

scratch

$-8x^{-3} \to 24x^{-4}$

$$= e^{\lim_{x\to\infty} \frac{\ln\left(1-\frac{8}{x^3}\right)}{1/x^3}} \qquad \frac{0}{0}$$

$\to x^{-3} \to -3x^{-4}$

$$\underset{L'H}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{1-8/x^3}\left(\frac{24}{\cancel{x^4}}\right)\left(\frac{-\cancel{x^4}}{3}\right)}{\frac{-3}{x^4}}}$$

FLIP
BACK
ALGEBRA

don't drop

$$= e^{\lim_{x\to\infty} \cancel{\frac{1}{1-8/x^3}}^{\,1} \cdot (-8)}$$

$$= \boxed{e^{-8}}$$

Match!

$0 \cdot (-\infty)$

3. $$\lim_{x\to 0^+} x^3 \cdot \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^3}} \quad \frac{-\infty}{\infty} \quad \overset{\text{L'H}}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-3}{x^4}} \quad \frac{-x^4}{3}$$

flip

$x^{-3} \to -3x^{-4}$

$$= \lim_{x\to 0^+} \frac{1}{\cancel{x}} \cdot \left(-\frac{x^{\cancel{4}\,3}}{3}\right) = \lim_{x\to 0^+} \frac{-\cancel{x^3}^{\,\to 0}}{3} = 0$$

1^∞

4. $$\lim_{x\to\infty} \left(1 - \arctan\left(\frac{3}{x^4}\right)\right)^{x^4} = e^{\lim_{x\to\infty} \ln\left[\left(1-\arctan\left(\frac{3}{x^4}\right)\right)^{x^4}\right]}$$

$\infty \cdot 0$

$$= e^{\lim_{x\to\infty} x^4 \cdot \ln\left(1-\arctan\left(\frac{3}{x^4}\right)\right)}$$

flip

$$= e^{\lim_{x\to\infty} \frac{\ln\left(1-\arctan\left(\frac{3}{x^4}\right)\right)}{\frac{1}{x^4}}} \quad \frac{0}{0}$$

$$= e^{\lim_{x\to\infty} \frac{\cancel{\frac{1}{1-\arctan\left(\frac{3}{x^4}\right)^{\to 0}}}^{\to 1} \cdot \left[\cancel{-\frac{1}{1+\left(\frac{3}{x^4}\right)^{2\,\to 0}}}^{\to 1}\right] \cdot \left(\frac{\cancel{-12}^{\,3}}{\cancel{x^5}}\right)}{\frac{\cancel{-}4}{\cancel{x^5}}}}$$

$$= e^{-3}$$

Match!