Quiz 1 Answers Fall 2023

1. $\int_4^{4\sqrt{3}} \frac{1}{\sqrt{64-x^2}} + \frac{1}{16+x^2}\,dx$ a-rules $= \arcsin\left(\frac{x}{8}\right) + \frac{1}{4}\arctan\left(\frac{x}{4}\right)\Big|_4^{4\sqrt{3}}$

$= \arcsin\left(\frac{4\sqrt{3}}{8}\right) + \frac{1}{4}\arctan\left(\frac{4\sqrt{3}}{4}\right) - \left(\arcsin\left(\frac{4}{8}\right) + \frac{1}{4}\arctan\left(\frac{4}{4}\right)\right)$

$= \arcsin\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{4}\arctan(\sqrt{3}) - \arcsin\left(\frac{1}{2}\right) - \frac{1}{4}\arctan(1)$

$= \frac{\pi}{3} + \frac{\pi}{12} - \frac{\pi}{6} - \frac{\pi}{16} = \frac{4\pi}{12} + \frac{\pi}{12} - \frac{2\pi}{12} - \frac{\pi}{16} = \frac{3\pi}{12} - \frac{\pi}{16} = \frac{4\pi}{16} - \frac{\pi}{16} = \frac{3\pi}{16}$

Match!

2. $\int_0^{\frac{1}{2}\ln\sqrt{3}} \frac{e^{2x}}{1+e^{4x}}\,dx = \int_0^{\frac{1}{2}\ln\sqrt{3}} \frac{e^{2x}}{1+(e^{2x})^2}\,dx = \frac{1}{2}\int_1^{\sqrt{3}} \frac{1}{1+u^2}\,du$

$u = e^{2x}$

$du = 2e^{2x}\,dx$

$\frac{1}{2}du = e^{2x}\,dx$

$x = 0 \Rightarrow u = e^0 = 1$

$x = \frac{1}{2}\ln\sqrt{3} \Rightarrow u = e^{2\left(\frac{1}{2}\ln\sqrt{3}\right)} = e^{\ln\sqrt{3}} = \sqrt{3}$

$= \frac{1}{2}\arctan u\Big|_1^{\sqrt{3}}$

$= \frac{1}{2}\left(\arctan\sqrt{3} - \arctan 1\right)$

$= \frac{1}{2}\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \frac{1}{2}\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \frac{1}{2}\left(\frac{\pi}{12}\right) = \frac{\pi}{24}$ Match!

3. $\int \frac{x^2+1}{x^2+1}\,dx$ slip-in/slip-out $= \int \frac{x^2+1-1}{x^2+1}\,dx$ split-split $= \int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\,dx = x - \arctan x + C$

4. $\int_1^e \frac{1}{x\sqrt{4-(\ln x)^2}}\,dx = \int_0^1 \frac{1}{\sqrt{4-u^2}}\,du = \arcsin\left(\frac{u}{2}\right)\Big|_0^1$ (a-rule)

$u = \ln x$
$du = \frac{1}{x}\,dx$

$x=1 \Rightarrow u = \ln 1 = 0$
$x=e \Rightarrow u = \ln e = 1$

$= \arcsin\left(\frac{1}{2}\right) - \arcsin 0$ ($\frac{1}{2} \to \frac{\pi}{6}$, $0 \to 0$)

$= \frac{\pi}{6} - 0 = \frac{\pi}{6}$

5. $\int_0^{\ln 3} \frac{1}{e^x(4-e^{-x})}\,dx = \int_3^{11/3} \frac{1}{u}\,du = \ln|u|\Big|_3^{11/3} = \ln\left|\frac{11}{3}\right| - \ln|3|$

$= \ln\left(\frac{11/3}{3}\right)$ ($\frac{1}{3}$)

$= \ln\left(\frac{11}{9}\right)$

$u = 4 - e^{-x}$
$du = -e^{-x}(-1)\,dx$
$= e^{-x}\,dx$
$du = \frac{1}{e^x}\,dx$

$x=0 \Rightarrow u = 4 - e^0 = 3$
$x=\ln 3 \Rightarrow u = 4 - e^{-\ln 3}$
$= 4 - e^{\ln(3^{-1})}$
$= 4 - \frac{1}{3} = \frac{11}{3}$