

Integration Reference Page

Math 121 D. Benedetto

Power Functions	<ul style="list-style-type: none"> • $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$ • $\int \frac{1}{x} dx = \ln x + C$
Trigonometric Functions	<ul style="list-style-type: none"> • $\int \sin x dx = -\cos x + C$ • $\int \cos x dx = \sin x + C$ • $\int \tan x dx = -\ln \cos x + C = \ln \sec x + C$ • $\int \sec x dx = \ln \sec x + \tan x + C$ • $\int \sec^2 x dx = \tan x + C$ • $\int \sec x \tan x dx = \sec x + C$
Exponentials and Logarithms	<ul style="list-style-type: none"> • $\int e^x dx = e^x + C$ • $\int e^{kx} dx = \frac{1}{k}e^{kx} + C$ • $\int \frac{1}{x} dx = \ln x + C$ • $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln ax+b + C$ • $\int \ln x dx = x \ln x - x + C$ using Integration By Parts?! <p style="text-align: center;">*****</p> <p>Review other e^x and $\ln x$ handout</p>

<p>Inverse Trig. Functions</p>	<ul style="list-style-type: none"> • $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C = \sin^{-1} x + C$ • $\int \frac{1}{1+x^2} dx = \arctan x + C = \tan^{-1} x + C$ • $\int \frac{1}{x\sqrt{x^2-1}} dx = \operatorname{arcsec} x + C = \sec^{-1} x + C$ • $\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C = \sin^{-1}\left(\frac{x}{a}\right) + C$ • $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$ • $\int \arcsin x dx = x \arcsin x + \sqrt{1-x^2} + C$ how?? • $\int \arctan x dx = x \arctan x - \frac{1}{2} \ln(1+x^2) + C$ how??
<p>Hyperbolic Functions</p> <p style="color: red;">ALL OPTIONAL</p>	<ul style="list-style-type: none"> • $\sinh x = \frac{e^x - e^{-x}}{2}$ • $\cosh x = \frac{e^x + e^{-x}}{2}$ • $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ • $\frac{d}{dx} \sinh x = \cosh x$ • $\frac{d}{dx} \cosh x = \sinh x$ • $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$ • $\int \sinh x dx = \cosh x + C$ • $\int \cosh x dx = \sinh x + C$ • $\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1} x + C$ Optional • $\int \frac{1}{1-x^2} dx = \tanh^{-1} x + C$ Optional • $\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C$ Optional

Products of Trig. Functions	<ul style="list-style-type: none"> • $\int \sin^m x \cos^n x dx = ??$ Know ODD/EVEN techniques • $\int \tan^m x \sec^n x dx = ??$ Know a few lower powered examples 												
Trig. Substitution	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center; border-bottom: 1px solid black;">Integrand Contains</th> <th style="text-align: center; border-bottom: 1px solid black;">Substitute</th> <th style="text-align: center; border-bottom: 1px solid black;">Identity</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">$\sqrt{a^2 - x^2}$</td> <td style="text-align: center;">$x = a \sin \theta$</td> <td style="text-align: center;">$\sin^2 \theta + \cos^2 \theta = 1$</td> </tr> <tr> <td style="text-align: center;">$\sqrt{a^2 + x^2}$</td> <td style="text-align: center;">$x = a \tan \theta$</td> <td style="text-align: center;">$\sec^2 \theta = 1 + \tan^2 \theta$</td> </tr> <tr> <td style="text-align: center;">$\sqrt{x^2 - a^2}$</td> <td style="text-align: center;">$x = a \sec \theta$</td> <td style="text-align: center;">$\tan^2 \theta = \sec^2 \theta - 1$</td> </tr> </tbody> </table>	Integrand Contains	Substitute	Identity	$\sqrt{a^2 - x^2}$	$x = a \sin \theta$	$\sin^2 \theta + \cos^2 \theta = 1$	$\sqrt{a^2 + x^2}$	$x = a \tan \theta$	$\sec^2 \theta = 1 + \tan^2 \theta$	$\sqrt{x^2 - a^2}$	$x = a \sec \theta$	$\tan^2 \theta = \sec^2 \theta - 1$
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Trigonometric Identities	<ul style="list-style-type: none"> • $\sin^2 \theta + \cos^2 \theta = 1$ • $\sec^2 \theta = 1 + \tan^2 \theta$ • $\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$ • $\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$ • $\sin(2\theta) = 2 \sin \theta \cos \theta$ <p style="text-align: center;">*****</p> <ul style="list-style-type: none"> • $\cosh^2 x - \sinh^2 x = 1$ 												
Integration by Parts	<ul style="list-style-type: none"> • $\int u dv = uv - \int v du$ • $\int f(x)g'(x) dx = f(x)g(x) - \int g(x)f'(x) dx$ • $\int_a^b u dv = uv _a^b - \int_a^b v du$ 												