Math 121, Section(s) 01, 02, Fall 2023

Homework #8

Due **THURSDAY, October 12 (NOTE CHANGE)** in Gradescope by 11:59 pm ET

Goal: Exploring Integrals using Secant Trig Subsitutions, some Complete the Square algebra and also Partial Fractions Decomposition (Proper cases).

FIRST: Read through and understand the following two Examples.

Recall: IF the denominator FACTORS, go use PFD; otherwise try Complete the Square algebra to start

Ex:

$$\int \frac{6}{x^2+2x+8}\,dx \overset{\text{complete square}}{=} \int \frac{6}{(x+1)^2+7}\,dx = 6\int \frac{1}{u^2+7}\,du$$

$$= 6\left(\frac{1}{\sqrt{7}}\right)\arctan\left(\frac{u}{\sqrt{7}}\right)+C = \boxed{\frac{6}{\sqrt{7}}\arctan\left(\frac{x+1}{\sqrt{7}}\right)+C}$$

$$\boxed{\begin{aligned} u &= x+1 \\ du &= dx \end{aligned}}$$

Ex:

$$\int_{-3}^{1} \frac{6}{x^2+2x-8}\,dx \overset{\text{factor}}{=} \int_{-3}^{1} \frac{6}{(x-2)(x+4)}\,dx \overset{\text{PFD}}{=} \int_{-3}^{1} \frac{1}{x-2}-\frac{1}{x+4}\,dx$$

$$= \ln|x-2| - \ln|x+4|\Big|_{-3}^{1} = \ln|-1| - \ln 5 - (\ln|-5| - \ln 1)$$

$$= 0 - \ln 5 - \ln 5 + 0 = \boxed{-2\ln 5}$$

Partial Fractions Decomposition:

$$\frac{6}{(x-2)(x+4)} = \frac{A}{x-2} + \frac{B}{x+4}$$

Clearing the denominator yields:

$6 = A(x+4) + B(x-2)$
$6 = Ax + 4A + Bx - 2B$
$6 = (A+B)x + (4A-2B)$
so that $A+B=0$ and $4A-2B=6$
Solve for $A=1$ and $B=-1$

Continue to NEXT Page for HW problems.

Compute each of the following Integrals. Simplify when possible.

1. $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}\,dx$

↑ sub $\boxed{\begin{array}{rl} x &= 3\sec\theta \\ dx &= 3\sec\theta\tan\theta d\theta \end{array}}$

↑ hint: $\dfrac{\tan^2\theta}{\sec^2\theta} = \sin^2\theta$ why?

2. $\displaystyle\int \frac{1}{x^2\sqrt{x^2-16}}\,dx$

↑ sub $\boxed{\begin{array}{rl} x &= 4\sec\theta \\ dx &= 4\sec\theta\tan\theta d\theta \end{array}}$

3. $\displaystyle\int \frac{1}{\sqrt{4-4x-x^2}}\,dx$

4. $\displaystyle\int_{-1}^{1} \frac{1}{x^2+4x+7}\,dx$

5. $\displaystyle\int \sqrt{3-2x-x^2}\,dx$

6. $\displaystyle\int \frac{x+4}{x^2+2x+5}\,dx$

7. $\displaystyle\int_3^5 \frac{6}{x^2-4x+7}\,dx$

8. $\displaystyle\int_0^3 \frac{6}{x^2-4x-5}\,dx$

9. $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$

10. $\displaystyle\int \frac{\arctan x}{x^2}\,dx = \int \arctan x \cdot \left(x^{-2}\right)\,dx$

11. $\displaystyle\int_{\ln 2}^{\ln 5} \frac{2e^x}{e^{2x}-1}\,dx$

12. $\displaystyle\int \frac{10}{(x-1)(x^2+9)}\,dx$

↑ hint: $\begin{aligned} \dfrac{2}{u^2-1} &= \dfrac{2}{(u-1)(u+1)} \\ &= \dfrac{A}{u-1} + \dfrac{B}{u+1} \end{aligned}$

↑ use PFD $\dfrac{10}{(x-1)(x^2+9)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+9}$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

7:30–9:00 pm TA Admire, SMUDD 206

9:00–10:30 pm TA Aidee, SMUDD 206

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Natalie, SMUDD 206

7:30–9:00 pm TA Gretta, SMUDD 206

9–10:30 pm TA Aidee, SMUDD 206

Wednesday: 1:00-3:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

9–10:30 pm TA Natalie, SMUDD 206

Thursday: none for Professor

6:00–7:30 pm TA Gretta, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

Friday: 12:00-2:00 pm

Please e-mail with questions/concerns: `dbenedetto@amherst.edu`

Either *Start early* or *don't start late.*

Thanks for working hard. I appreciate it!