Math 121, Section(s) 01, 02, Fall 2023

Homework #7

Due **Friday, September 29th** in Gradescope by 11:59 pm ET

Goal: Exploring Trigonometric Integrals and Trigonometric Substitution

FIRST: Read through and understand the following two Examples.

Ex:

$$\int \frac{1}{[1+x^2]^{\frac{7}{2}}}\,dx = \int \frac{1}{(1+\tan^2\theta)^{\frac{7}{2}}}\cdot \sec^2\theta\,d\theta = \int \frac{1}{(\sec^2\theta)^{\frac{7}{2}}}\cdot \sec^2\theta\,d\theta$$

$$= \int \frac{1}{(\sqrt{\sec^2\theta})^7}\cdot \sec^2\theta\,d\theta = \int \frac{1}{(\sec\theta)^7}\cdot \sec^2\theta\,d\theta$$

$$= \int \frac{\sec^2\theta}{\sec^7\theta}\,d\theta = \int \frac{1}{\sec^5\theta}\,d\theta$$

$$= \int \cos^5\theta\,d\theta = \int \cos^4\theta\cos\theta\,d\theta$$

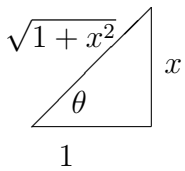
$$= \int (1-\sin^2\theta)^2\cos\theta\,d\theta = \int (1-w^2)^2\,dw$$

$$= \int 1-2w^2+w^4\,dw = w-\frac{2w^3}{3}+\frac{w^5}{5}+C$$

$$= \sin\theta-\frac{2\sin^3\theta}{3}+\frac{\sin^5\theta}{5}+C$$

$$= \boxed{\frac{x}{\sqrt{1+x^2}}-\frac{2}{3}\left(\frac{x}{\sqrt{1+x^2}}\right)^3+\frac{1}{5}\left(\frac{x}{\sqrt{1+x^2}}\right)^5+C}$$

Trig. Sub $\boxed{\begin{array}{l} x=\tan\theta \\ \\ dx=\sec^2\theta d\theta \end{array}}$



$\boxed{\begin{array}{l} w=\sin\theta \\ \\ dw=\cos\theta\,d\theta \end{array}}$

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Ex:

$$\int \frac{x^2}{\sqrt{4-x^2}}\,dx = \int \frac{(2\sin\theta)^2}{\sqrt{4-4\sin^2\theta}}\,2\cos\theta\,d\theta = \int \frac{4\sin^2\theta}{\sqrt{4(1-\sin^2\theta)}}\,2\cos\theta\,d\theta$$

$$= 4\int \frac{\sin^2\theta}{\sqrt{4}\sqrt{\cos^2\theta}}\,2\cos\theta\,d\theta = 4\int \frac{\sin^2\theta}{2\cos\theta}\,2\cos\theta\,d\theta$$

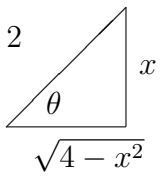
$$= 4\int \sin^2\theta\,d\theta = 4\int \frac{1-\cos(2\theta)}{2}\,d\theta$$

$$= 2\int 1-\cos(2\theta)\,d\theta = 2\left(\theta - \frac{\sin(2\theta)}{2}\right) + C$$

$$= 2\left(\theta - \frac{2\sin\theta\cos\theta}{2}\right) + C = 2\left(\theta - \sin\theta\cos\theta\right) + C$$

$$= \boxed{2\left[\arcsin\left(\frac{x}{2}\right) - \left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right)\right] + C}$$

Trig. Substitute $\boxed{\begin{array}{l} x = 2\sin\theta \\ \\ dx = 2\cos\theta d\theta \end{array}}$



Compute each of the following Integrals. Simplify.

1. $\int \sin^2 x\ \cos^3 x\ dx$
2. $\int_0^{\frac{\pi}{2}} \sin^5 x\ dx$
3. $\int_0^{\frac{\pi}{2}} \cos^2\theta\ d\theta$
4. $\int_0^{\frac{\pi}{2}} \sin^2 x\ \cos^2 x\ dx$
5. $\int x\ \sin^2 x\ dx$
6. $\int_0^1 x^3\sqrt{1-x^2}\ dx$ use Trig Sub
7. $\int \sqrt{9-x^2}\ dx$
8. $\int \frac{1}{(4+x^2)^{\frac{5}{2}}}\ dx$
9. $\int x\ \arcsin x\ dx$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

7:30–9:00 pm TA Admire, SMUDD 206

9:00–10:30 pm TA Aidee, SMUDD 206

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Natalie, SMUDD 206

7:30–9:00 pm TA Gretta, SMUDD 206

9–10:30 pm TA Aidee, SMUDD 206

Wednesday: 1:00-3:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

9–10:30 pm TA Natalie, SMUDD 206

Thursday: none for Professor

6:00–7:30 pm TA Gretta, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

Friday: 12:00-2:00 pm

This is the end of exam #1 material

Go to office hours and also you're welcome at Math Fellow TA hours