

Homework #6

Due Wednesday, September 27th in Gradescope by 11:59 pm ET

Goal: Exploring Limits using L'Hôpital's Rule, and Integrals using Integration By Parts

FIRST: Read through and understand the following two Examples.

$$\begin{aligned}
 \text{Ex: } \lim_{x \rightarrow \infty} \left(e^{\frac{1}{x}} - \frac{4}{x} \right)^x &\stackrel{1^\infty}{=} \lim_{x \rightarrow \infty} e^{\ln \left(\left(e^{\frac{1}{x}} - \frac{4}{x} \right)^x \right)} = e^{\lim_{x \rightarrow \infty} \ln \left(\left(e^{\frac{1}{x}} - \frac{4}{x} \right)^x \right)} = e^{\lim_{x \rightarrow \infty} x \ln \left(e^{\frac{1}{x}} - \frac{4}{x} \right)} \\
 &\stackrel{\infty \cdot 0}{=} e^{\lim_{x \rightarrow \infty} \frac{\ln \left(e^{\frac{1}{x}} - \frac{4}{x} \right)}{\frac{1}{x}}} \stackrel{\left(\frac{0}{0}\right)^{\text{L'H}}}{=} e^{\lim_{x \rightarrow \infty} \frac{\left(\frac{1}{e^{\frac{1}{x}} - \frac{4}{x}} \right) \cdot \left[e^{\frac{1}{x}} \left(-\frac{1}{x^2} \right) + \frac{4}{x^2} \right]}{-\frac{1}{x^2}}} \\
 &= e^{\lim_{x \rightarrow \infty} \left(\frac{1}{e^{\frac{1}{x}} - \frac{4}{x}} \right) \cdot \left[e^{\frac{1}{x}} \left(-\frac{1}{x^2} \right) + \frac{4}{x^2} \right]} \stackrel{(-x^2)}{=} e^{\lim_{x \rightarrow \infty} \left(\frac{1}{e^{\frac{1}{x}} - \frac{4}{x}} \right) \cdot \left(e^{\frac{1}{x}} (1) - 4 \right)} = \boxed{e^{-3}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex: } \int \arctan \left(\frac{1}{x} \right) dx &= x \arctan \left(\frac{1}{x} \right) + \int \frac{x}{x^2 + 1} dx = x \arctan \left(\frac{1}{x} \right) + \frac{1}{2} \int \frac{1}{w} dw \\
 &= x \arctan \left(\frac{1}{x} \right) + \frac{1}{2} \ln |w| + C = \boxed{x \arctan \left(\frac{1}{x} \right) + \frac{\ln |x^2 + 1|}{2} + C}
 \end{aligned}$$

IBP	$u = \arctan \left(\frac{1}{x} \right) \qquad dv = 1 dx$	$w\text{-sub } \begin{cases} w = x^2 + 1 \\ dw = 2x dx \\ \frac{1}{2} dw = x dx \end{cases}$
	$du = \frac{1}{1 + \left(\frac{1}{x} \right)^2} \cdot \left(-\frac{1}{x^2} \right) dx \qquad v = x$	
	$du = -\frac{1}{x^2 + 1} dx \qquad \leftarrow \text{simplify}$	

Continue to NEXT Page for HW problems.

Compute each of the following Limits. Simplify.

1. $\lim_{x \rightarrow \infty} \frac{\ln(5 + e^{3x})}{x}$

2. $\lim_{x \rightarrow \infty} \left(\frac{x}{x+1} \right)^x$

3. $\lim_{x \rightarrow \infty} \left(e^{\frac{1}{x^6}} - \frac{6}{x^6} \right)^{x^6}$

Compute each of the following Integrals using Integration by Parts. Simplify.

4. $\int x \cos(5x) dx$

5. $\int_0^1 \arctan x dx$

6. $\int_0^5 \frac{x^2}{e^x} dx$

7. $\int (\ln x)^2 dx$

8. $\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx$

9. $\int x \arctan x dx$

10. $\int \ln(x^2 + 7) dx$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

7:30–9:00 pm TA Admire, SMUDD 206

9:00–10:30 pm TA Aidee, SMUDD 206

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Natalie, SMUDD 206

7:30–9:00 pm TA Gretta, SMUDD 206

9–10:30 pm TA Aidee, SMUDD 206

Wednesday: 1:00-3:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

9–10:30 pm TA Natalie, SMUDD 206

Thursday: none for Professor

6:00–7:30 pm TA Gretta, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

Friday: 12:00-2:00 pm

Do something different or new this week, for studying purposes.

Go to office hours and also go to Math Fellow TA hours