Math 121, Section(s) 01, 02, Fall 2023

Homework #5

Due **Friday, September 22nd** in Gradescope by 11:59 pm ET

Goal: Exploring Limits using L'Hôpital's Rule, and solidifying size arguments

FIRST: Read through and understand the following two Limit Examples.

$$\text{Ex}: \lim_{x\to 0}\frac{\arcsin x+\cos(3x)-e^x}{\arctan(3x)+x^2-\sin(3x)} \overset{\left(\frac{0}{0}\right)}{\overset{\text{L'H}}{=}} \lim_{x\to 0}\frac{\frac{1}{\sqrt{1-x^2}}-3\sin(3x)-e^x}{\frac{1}{1+(3x)^2}\cdot(3)+2x-3\cos(3x)} \overset{\left(\frac{0}{0}\right)}{\overset{\text{prep}}{=}} \lim_{x\to 0}\frac{(1-x^2)^{-\frac{1}{2}}-3\sin(3x)-e^x}{3\left(1+9x^2\right)^{-1}+2x-3\cos(3x)}$$

$$\overset{\left(\frac{0}{0}\right)}{\overset{\text{L'H}}{=}} \lim_{x\to 0}\frac{-\frac{1}{2}\left(1-x^2\right)^{-\frac{3}{2}}\cdot(-2x)-9\cos(3x)-e^x}{-3\left(1+9x^2\right)^{-2}\cdot(18x)+2+9\sin(3x)} \overset{\text{rewrite}}{=} \lim_{x\to 0}\frac{\frac{x}{(1-x^2)^{\frac{3}{2}}}\to 0-9\cos(3x)\to 1-e^x\to 1}{\frac{-54x}{(1+9x^2)^2}\to 0+2+9\sin(3x)\to 0}$$

$$= \frac{-9-1}{2} = \frac{-10}{2} = \boxed{-5}$$

$$\text{Ex: } \lim_{x\to\infty}\left(1-\frac{2}{x^3}\right)^{x^3} \overset{1^\infty}{=} \lim_{x\to\infty} e^{\ln\left(\left(1-\frac{2}{x^3}\right)^{x^3}\right)} \overset{\infty\cdot 0}{=} e^{\lim_{x\to\infty} x^3\ln\left(1-\frac{2}{x^3}\right)}$$

$$= e^{\lim_{x\to\infty}\frac{\ln\left(1-\frac{2}{x^3}\right)}{\frac{1}{x^3}}} \overset{\left(\frac{0}{0}\right)^{\text{L'H}}}{=} e^{\lim_{x\to\infty}\frac{\left(\frac{1}{1-\frac{2}{x^3}}\right)\left(\frac{6}{x^4}\right)}{-\frac{3}{x^4}}}$$

$$= e^{\lim_{x\to\infty}\left(\frac{1}{1-\frac{2}{x^3}}\right)\left(\frac{6}{x^4}\right)\cdot\left(-\frac{x^4}{3}\right)} = e^{\lim_{x\to\infty}\left(\frac{1}{1-\frac{2}{x^3}\to 0}\right)(-2)} = e^{1\cdot(-2)} = \boxed{e^{-2}}$$

Continue to NEXT Page for HW problems.

Compute each of the following Limits. Simplify. *Justify every step.*

1. $\lim_{\theta \to \frac{\pi}{2}} \dfrac{1-\sin\theta}{1+\cos(2\theta)}$

2. $\lim_{x \to \infty} \dfrac{\ln x}{\sqrt{x}}$

3. $\lim_{x \to 0^+} \dfrac{\ln x}{x}$

4. $\lim_{x \to 0} \dfrac{e^{2x}-1-2x}{x^2}$

5. $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$

6. $\lim_{x \to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$

7. $\lim_{x \to 0} \dfrac{\arcsin(3x)}{\arctan(4x)}$

8. $\lim_{x \to 0} \dfrac{x - \arcsin x}{\arctan(2x) - 2x}$

9. $\lim_{x \to 0} \dfrac{3xe^x - \arctan(3x)}{x + \ln(1-x)}$

10. $\lim_{x \to 0} \dfrac{\arcsin x + x^2 - x}{\cos x - \arctan(5x) - e^{-5x}}$

11. $\lim_{x \to \infty} x \sin\left(\dfrac{\pi}{x}\right)$

12. $\lim_{x \to \infty} x \ln\left(1 - \dfrac{1}{x}\right)$

13. $\lim_{x \to 0^+} x \ln x$

14. $\lim_{x \to 0^+} \sqrt{x} \ln x$

15. $\lim_{x \to \infty} x^2 e^{-x}$

16. $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$

17. $\lim_{x \to 0^+} (1 + \ln(1-3x))^{\frac{1}{x}}$

18. $\lim_{x \to \infty} \left(1 - \arctan\left(\dfrac{7}{x^4}\right)\right)^{x^4}$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

7:30–9:00 pm TA Admire, SMUDD 206

9:00–10:30 pm TA Aidee, SMUDD 206

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Natalie, SMUDD 206

7:30–9:00 pm TA Gretta, SMUDD 206

9–10:30 pm TA Aidee, SMUDD 206

Wednesday: 1:00-3:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

9–10:30 pm TA Natalie, SMUDD 206

Thursday: none for Professor

6:00–7:30 pm TA Gretta, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

Friday: 12:00-2:00 pm

Please e-mail with questions/concerns: `dbenedetto@amherst.edu`

Start early. Box your answers.

Show all details and justifications in a nice final draft. No Mess.