

Homework #16

Due **Wednesday, November 15th** in Gradescope by 11:59 pm ET

Goal: Exploring more of the Relationship between Power Series and functions, including Differentiation and Integration of Power Series. Also *substitution* into a known MacLaurin Series. Also SUMS which are not Geometric.

Find the Series Representation for the following functions using *substitution* and determine the Radius of Convergence R . Simplify.

$$1. \frac{1}{1+x^2} \quad 2. \frac{x^2}{x^4+16} \quad 3. x^3 \cos(x^2) \quad 4. 5x^2 \sin(5x)$$

$$5. \frac{d}{dx}(x^3 \arctan(7x)) \quad 6. \int x^3 \arctan(7x) dx \quad 7. \frac{d}{dx} x^2 \ln(1+6x) \quad 8. \int x^4 e^{-x^3} dx$$

9. Find the Series Representation for $f(x) = \frac{1}{(1+x)^2}$

Hint: $\frac{1}{(1+x)^2} = \frac{d}{dx} \left(-\frac{1}{1+x} \right) \stackrel{PS?}{=} \dots$

10. Prove the Power Series Representation formula for $\arctan x$, as shown in class. Yes, show that $C = 0$.

11. Find Series Representation for $\ln(5-x)$. Solve for C and the Radius R .

Hint: $\ln(5-x) = \int \frac{-1}{5-x} dx = \int \frac{-1}{5\left(1-\frac{x}{5}\right)} dx = -\frac{1}{5} \int \frac{1}{1-\frac{x}{5}} dx \stackrel{PS?}{=} \dots$

12. Find the MacLaurin Series for $f(x) = e^{-2x}$ using two different methods.

First, using the *Definition* of the MacLaurin Series (“Chart Method”).

Second, use Substitution into a known series. Your answers should be in Sigma notation.

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13. You do **not** need to state the Radius. Answers should be in Sigma notation $\sum_{n=0}^{\infty}$ here.

You may use the fact that $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ without extra justification.

(a) Use the Definition (“Chart Method”) to compute the MacLaurin Series for $F(x) = \cos x$.

(b) Use Differentiation to compute the Series for $F(x) = \cos x$.

(c) Use Integration to compute the Series for $F(x) = \cos x$.

Hints: yes, you should solve for $+C$. yes, C should equal 1. Show why $C = 1$.

Find the Sum of each of the following Series, which do converge.

14. $\sum_{n=0}^{\infty} \frac{7^n}{n!}$

15. $\sum_{n=0}^{\infty} \frac{(-1)^n 5^n}{n!}$

16. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$

17. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n)!}$

18. $\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$

19. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

20. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \dots$

21. $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \dots$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

7:30–9:00 pm TA Admire, SMUDD 206

9:00–10:30 pm TA Aidee, SMUDD 206

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Natalie, SMUDD 206

7:30–9:00 pm TA Gretta, SMUDD 206

9–10:30 pm TA Aidee, SMUDD 206

Wednesday: 1:00–3:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

9–10:30 pm TA Natalie, SMUDD 206

Thursday: none for Professor

6:00–7:30 pm TA Gretta, SMUDD 206

7:30–9:00 pm TA James, SMUDD 206

Friday: 12:00–2:00 pm

Pay careful attention to details here.

Manipulating power series requires a balance
of memory and technical skill.