Math 121 Exam 1
October 4, 2023

● This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

● Numerical answers such as $\sin\left(\dfrac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\arctan(\sqrt{3})$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

● Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. Limits [32 Points total]   Evaluate each of the following. Please justify/simplify.

(a) Show that $\displaystyle\lim_{x\to 0}\ \frac{\ln(1+5x)-5x}{\arcsin(3x)+e^{-3x}-1}=\boxed{-\dfrac{25}{9}}$

(b) Show that $\displaystyle\lim_{x\to 0^+}\ x^3\ln x=\boxed{0}$

(c) Show that $\displaystyle\lim_{x\to\infty}\ \left(1-\arctan\left(\frac{3}{x^4}\right)\right)^{x^4}=\boxed{e^{-3}}$

Integrals [36 Points total]   Compute each of the following Definite Integrals. Justify.

2. Show that $\displaystyle\int_{-3}^{3}\sqrt{9-x^2}\ dx=\boxed{\dfrac{9\pi}{2}}$

3. Show that $\displaystyle\int_{0}^{\ln\sqrt{3}}\frac{e^{2x}}{3+e^{4x}}\ dx=\boxed{\dfrac{\pi}{12\sqrt{3}}}$

4. Show that $\displaystyle\int_{1}^{e}x^3\ln x\ dx=\boxed{\dfrac{1+3e^4}{16}}$

More Integrals [32 Points total]   Compute each of the following Indefinite Integrals. Justify.

5. Compute $\displaystyle\int x^2\arcsin x\ dx$

6. Compute $\displaystyle\int \frac{x}{(4+x^2)^{\frac{7}{2}}}\ dx$ using a Trig Substitution   Hint: $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$