Integration Reference Page

Math 121 D. Benedetto

Power Functions	• $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C \quad \text{for} \quad n \neq -1$ • $\int \dfrac{1}{x} \, dx = \ln \lvert x \rvert + C$
Trigonometric Functions	• $\int \sin x \, dx = -\cos x + C$ • $\int \cos x \, dx = \sin x + C$ • $\int \tan x \, dx = -\ln \lvert \cos x \rvert + C = \ln \lvert \sec x \rvert + C$ • $\int \sec x \, dx = \ln \lvert \sec x + \tan x \rvert + C$ • $\int \sec^2 x \, dx = \tan x + C$ • $\int \sec x \tan x \, dx = \sec x + C$
Exponentials and Logarithms	• $\int e^x \, dx = e^x + C$ • $\int e^{kx} \, dx = \dfrac{1}{k} e^{kx} + C$ • $\int \dfrac{1}{x} \, dx = \ln \lvert x \rvert + C$ • $\int \dfrac{1}{ax+b} \, dx = \dfrac{1}{a} \ln \lvert ax+b \rvert + C$ • $\int \ln x \, dx = x \ln x - x + C$ using Integration By Parts?! ********** Review other e^x and $\ln x$ handout

Inverse Trig. Functions	• $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C = \sin^{-1} x + C$ • $\int \frac{1}{1+x^2}\,dx = \arctan x + C = \tan^{-1} x + C$ • $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \text{arcsec} x + C = \sec^{-1} x + C$ • $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C = \sin^{-1}\left(\frac{x}{a}\right) + C$ • $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$ • $\int \arcsin x\,dx = x\arcsin x + \sqrt{1-x^2} + C$ how?? • $\int \arctan x\,dx = x\arctan x - \frac{1}{2}\ln(1+x^2) + C$ how??
Hyperbolic Functions	• $\sinh x = \frac{e^x - e^{-x}}{2}$ • $\cosh x = \frac{e^x + e^{-x}}{2}$ • $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ • $\frac{d}{dx}\sinh x = \cosh x$ • $\frac{d}{dx}\cosh x = \sinh x$ • $\frac{d}{dx}\tanh x = \text{sech}^2 x$ • $\int \sinh x\,dx = \cosh x + C$ • $\int \cosh x\,dx = \sinh x + C$ • $\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1} x + C$ Optional • $\int \frac{1}{1-x^2}\,dx = \tanh^{-1} x + C$ Optional • $\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1} x + C$ Optional

<table>
<tr><td>Products of Trig. Functions</td><td>• $\int \sin^m x \cos^n x \; dx = ??$ Know ODD/EVEN techniques
• $\int \tan^m x \sec^n x \; dx = ??$ Know a few lower powered examples</td></tr>
<tr><td>Trig. Substitution</td><td>
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<tr><th>Integrand Contains</th><th>Substitute</th><th>Identity</th></tr>
<tr><td>$\sqrt{a^2 - x^2}$</td><td>$x = a \sin\theta$</td><td>$\sin^2\theta + \cos^2\theta = 1$</td></tr>
<tr><td>$\sqrt{a^2 + x^2}$</td><td>$x = a \tan\theta$</td><td>$\sec^2\theta = 1 + \tan^2\theta$</td></tr>
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</td></tr>
<tr><td>Trigonometric Identities</td><td>• $\sin^2\theta + \cos^2\theta = 1$
• $\sec^2\theta = 1 + \tan^2\theta$
• $\sin^2\theta = \dfrac{1 - \cos(2\theta)}{2}$
• $\cos^2\theta = \dfrac{1 + \cos(2\theta)}{2}$
• $\sin(2\theta) = 2\sin\theta\cos\theta$
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• $\cosh^2 x - \sinh^2 x = 1$</td></tr>
<tr><td>Integration by Parts</td><td>• $\int u \; dv = uv - \int v \; du$
• $\int f(x)g'(x) \; dx = f(x)g(x) - \int g(x)f'(x) \; dx$
• $\int_a^b u \; dv = uv\big|_a^b - \int_a^b v \; du$</td></tr>
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