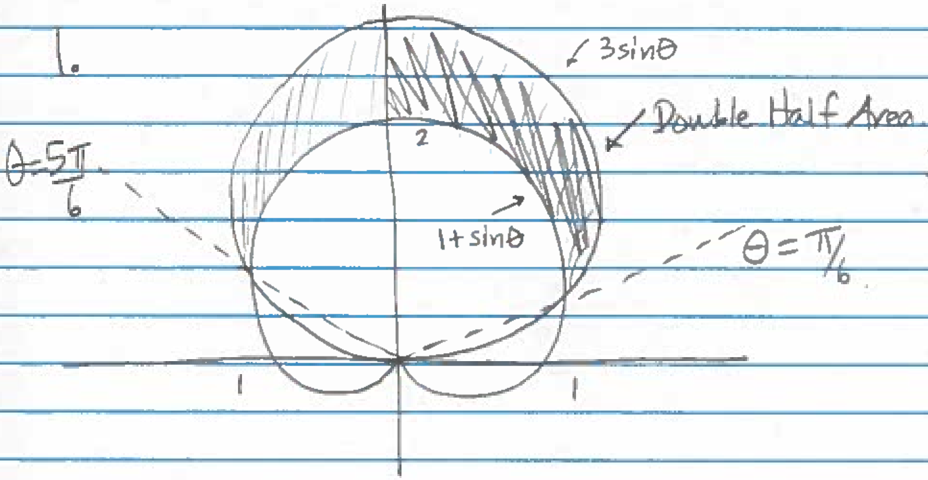
Self-Assessment Quiz #12 Answers

1.



Intersect? $3\sin\theta = 1+\sin\theta$

$2\sin\theta = 1$

$\sin\theta = 1/2$

$\Rightarrow \theta = \pi/6,\ 5\pi/6$

$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (r_{outer})^2 - (r_{inner})^2\, d\theta = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (3\sin\theta)^2 - \overset{(1+2\sin\theta+\sin^2\theta)}{(1+\sin\theta)^2}\, d\theta$$

$$= \cancel{2}\left(\frac{1}{\cancel{2}}\int_{\pi/6}^{\pi/2} 9\sin^2\theta - 1 - 2\sin\theta - \sin^2\theta\, d\theta\right)$$

Double using symmetry

$$= \int_{\pi/6}^{\pi/2} \overset{8\left(\frac{1-\cos(2\theta)}{2}\right)}{\cancel{8\sin^2\theta}} - 1 - 2\sin\theta\, d\theta$$

$$= \int_{\pi/6}^{\pi/2} \underbrace{4 - 4\cos(2\theta) - 1}_{3} - 2\sin\theta\, d\theta$$

$$= 3\theta - 2\sin(2\theta) + 2\cos\theta \Big|_{\pi/6}^{\pi/2}$$

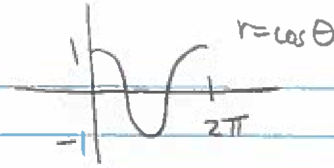
$$= 3\frac{\pi}{2} - \overset{0}{\cancel{2\sin\pi}} + \overset{0}{\cancel{2\cos\frac{\pi}{2}}} - \left(\overset{\pi/2}{\cancel{3\left(\frac{\pi}{6}\right)}} - \underbrace{\overset{\sqrt{3}/2}{\cancel{2\sin\left(\frac{\pi}{3}\right)}}}_{-\sqrt{3}} + \underbrace{\overset{\sqrt{3}/2}{\cancel{2\cos\left(\frac{\pi}{6}\right)}}}_{+\sqrt{3}}\right)$$

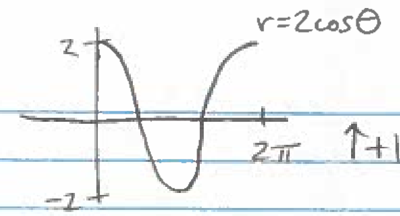
cancel

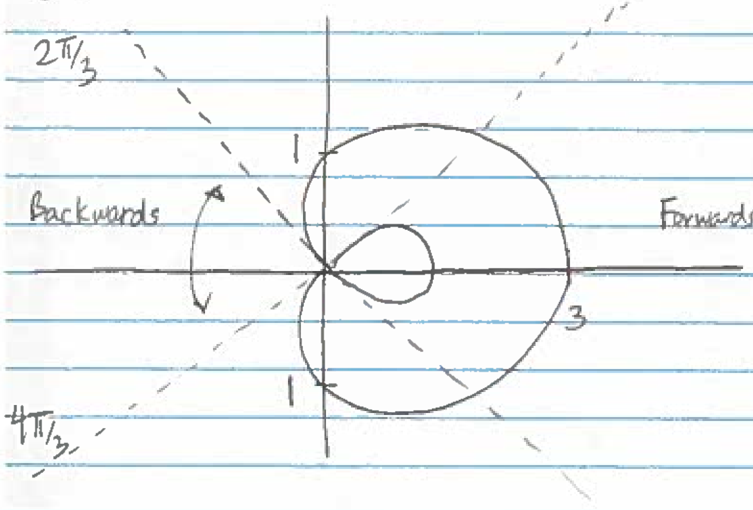
$$= \frac{3\pi}{2} - \frac{\pi}{2} = \boxed{\pi}$$

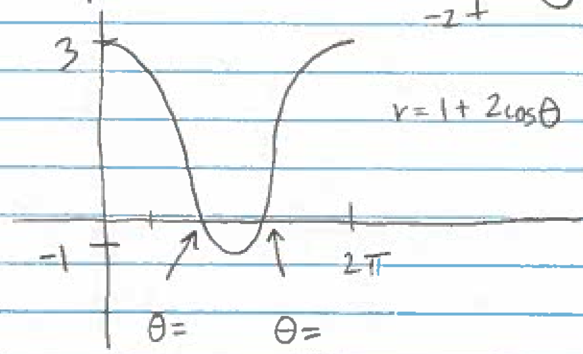
2. $r = 1 + 2\cos\theta$

(a)









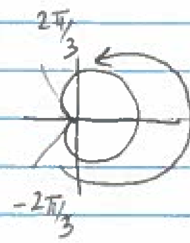
Set $1 + 2\cos\theta = 0$

Solve $\cos\theta = -1/2$

$\cos\theta = 1/2$ when $\theta = \pi/3$

$\Rightarrow \cos\theta = -1/2$ for $\theta = 2\pi/3, 4\pi/3$

(b) Larger Loop θ ranges from $-2\pi/3$ to $2\pi/3$

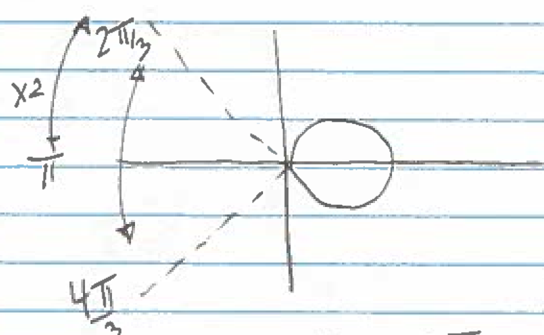


$$A = \frac{1}{2}\int_{-2\pi/3}^{2\pi/3} (1 + 2\cos\theta)^2\, d\theta$$

$$= \cancel{2}\left[\frac{1}{\cancel{2}}\int_0^{2\pi/3} (1 + 2\cos\theta)^2\, d\theta\right]$$

Double using Symmetry

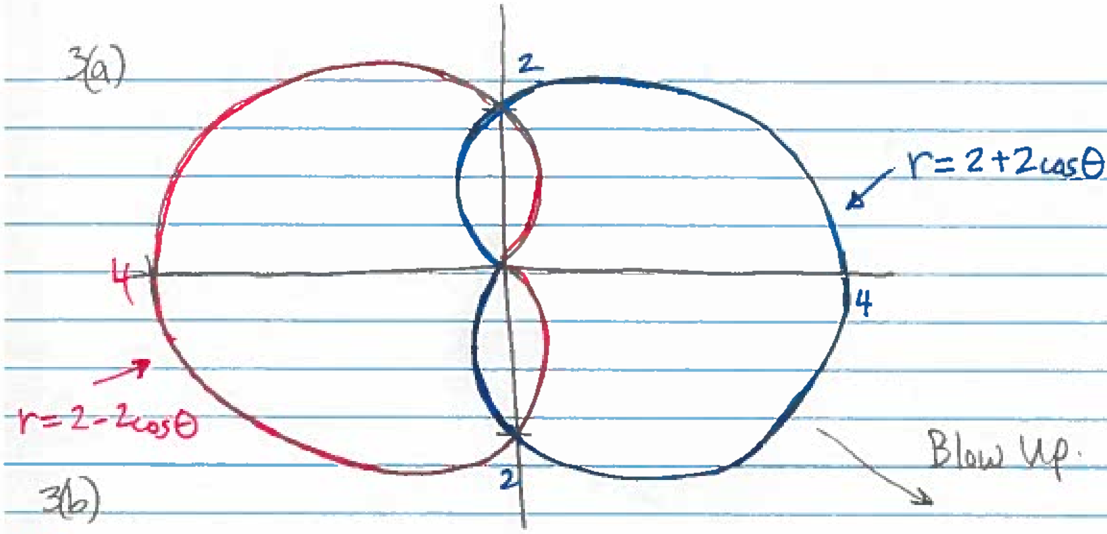
(c) Smaller Loop



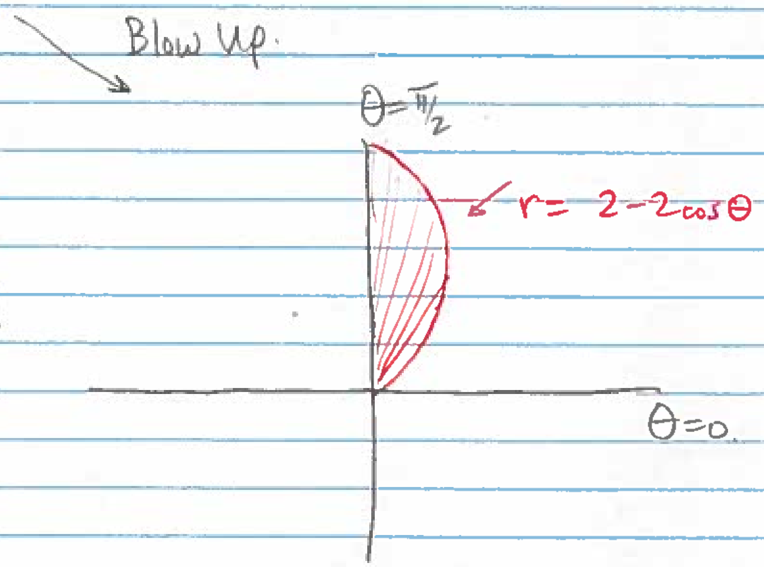
$$A = \frac{1}{2}\int_{2\pi/3}^{4\pi/3} (1+2\cos\theta)^2\, d\theta = \cancel{2}\left[\frac{1}{\cancel{2}}\int_{2\pi/3}^{\pi} (1 + 2\cos\theta)^2\, d\theta\right]$$

Double using Symmetry

3(a)

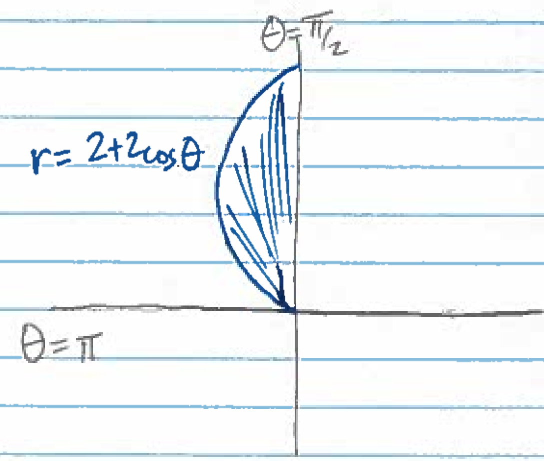


3(b)



$$\circledast\quad A = 4\left[\frac{1}{2}\int_0^{\pi/2}(2-2\cos\theta)^2\,d\theta\right]$$

using Symmetry
4 copies of one
quarter bulb.



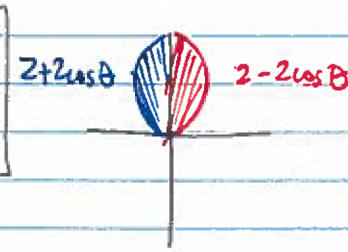
OR $$= 4\left[\frac{1}{2}\int_{\pi/2}^{\pi}(2+2\cos\theta)^2\,d\theta\right]$$

OR $$= 4\left[\frac{1}{2}\int_{\pi}^{3\pi/2}(2+2\cos\theta)^2\,d\theta\right]$$

OR $$= 4\left[\frac{1}{2}\int_{3\pi/2}^{2\pi}(2-2\cos\theta)^2\,d\theta\right]$$

OR $$= 2\left[\frac{1}{2}\int_0^{\pi/2}(2-2\cos\theta)^2\,d\theta + \frac{1}{2}\int_{\pi/2}^{\pi}(2+2\cos\theta)^2\,d\theta\right]$$

Double Top Bubble.



(*) Finish First Integral

$$A = \cancel{4}\left[\frac{1}{\cancel{2}}\int_0^{\pi/2}(2-2\cos\theta)^2\,d\theta\right] = 2\int_0^{\pi/2} 4 - 8\cos\theta + 4\cos^2\theta\,d\theta$$

$$= 2\int_0^{\pi/2} 4 - 8\cos\theta + \underbrace{4\left(\frac{1+\cos(2\theta)}{2}\right)}_{2+2\cos(2\theta)}\,d\theta = 2\int_0^{\pi/2} 6 - 8\cos\theta + 2\cos(2\theta)\,d\theta$$

combine.

$$= 2\left[6\theta - 8\sin\theta + \frac{\sin(2\theta)}{2}\right]\Bigg|_0^{\pi/2}$$

$$= 2\left[6\left(\frac{\pi}{2}\right) - 8\cancel{\sin}^{1}\left(\frac{\pi}{2}\right) + \cancel{\frac{\sin\pi}{2}}^{0} - \left(0 - 8\cancel{\sin 0}^{0} + \cancel{\frac{\sin 0}{2}}^{0}\right)\right]$$

$$= 2\left[3\pi - 8\right] = \boxed{6\pi - 16}$$