

**Homework #3**Due **Wednesday, September 14th** in Gradescope by 11:59 pm ET**Goal:** Solidifying Calculus for Inverse Sine and Inverse Tangent.**FIRST:** Read through and understand the following two Derivative proofs.Ex: **PROVE** that  $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$ Proof: Let  $y = \arctan x$       Looking to solve for  $\frac{dy}{dx}$ Invert  $\tan y = x$ Differentiate  $\frac{d}{dx} (\tan y) = \frac{d}{dx} (x)$ 

$$\sec^2 y \cdot \frac{dy}{dx} = 1$$

Solve  $\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + (\tan y)^2} = \frac{1}{1 + x^2}$ Ex: **PROVE** that  $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$ Proof: Let  $y = \arcsin x$       Looking to solve for  $\frac{dy}{dx}$ Invert  $\sin y = x$ Differentiate  $\frac{d}{dx} (\sin y) = \frac{d}{dx} (x)$ 

$$\cos y \cdot \frac{dy}{dx} = 1$$

Solve  $\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - \sin^2 y}} = \frac{1}{\sqrt{1 - (\sin y)^2}} = \frac{1}{\sqrt{1 - x^2}}$ 

Continue to NEXT Page for HW problems.

Differentiate the following functions. Simplify.

1.  $f(x) = \tan^{-1}(x^2)$       2.  $f(x) = (\tan^{-1}(x))^2$   
3.  $y = x \sin^{-1} x + \sqrt{1-x^2}$       4.  $f(x) = \ln\left(1 - \arcsin\left(\frac{2}{x^4}\right)\right)$

5. Find the value of the expression  $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

6. Simplify the expression  $\sin(\tan^{-1} x)$

7. Compute the Second Derivative for  $f(x) = \arctan(2x)$

8. Compute the Second Derivative for  $f(x) = \arcsin(6x)$

9. **Prove** that  $\frac{d}{dx} \sin^{-1}(3x) = \frac{3}{\sqrt{1-9x^2}}$

10. **Prove** that  $\frac{d}{dx} \tan^{-1}(5x) = \frac{5}{1+25x^2}$

11. Use Integration to **Justify** that  $\int \frac{1}{3+x^2} dx = \frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) + C$

Compute each of the following Integrals. Simplify.

12.  $\int \frac{x^2}{x^2+1} dx$

13.  $\int \frac{x+1}{x^2+1} dx$

14.  $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2} dx$

15.  $\int_0^{\frac{1}{2}} \frac{\arcsin x}{\sqrt{1-x^2}} dx$

16.  $\int \frac{1}{\sqrt{1-x^2} \cdot \sin^{-1} x} dx$

17.  $\int_1^3 \frac{1}{\sqrt{x}(1+x)} dx$

18.  $\int_0^{\ln 3} \frac{e^x}{1+e^x} dx$

19.  $\int_0^{\frac{1}{2} \ln 3} \frac{e^x}{1+e^{2x}} dx$

20.  $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}} dx$

21.  $\int_3^{3\sqrt{3}} \frac{1}{\sqrt{36-x^2}} + \frac{1}{9+x^2} dx$

# REGULAR OFFICE HOURS

**Monday: 12:00–3:00 pm**

7:30–9:00 pm TA Aidee, SMUDD 206

9:00–10:30 pm TA Mia, SMUDD 206

**Tuesday: 1:00–4:00 pm**

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Karime, SMUDD 206

9–10:30 pm TA Ali, SMUDD 206

**Wednesday: 1:00–3:00 pm**

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Ali, SMUDD 206

**Thursday: none for Professor**

7:30–9:00 pm TA Aidee, SMUDD 206

9–10:30 pm TA Karime, SMUDD ???

**Friday: 12:00–2:00 pm**

- Please do not wait until the last night to start.
- Please stop by for help! Please try the homework before you come by though. Final Answer keys are posted on the webpage. Please do **not** look at them unless you have completed the problems. **They are not a replacement for my help or your understanding.**
- You can also find help at the Math Fellow (Mia, Aidee, Karime, Ali or Admire) sessions.