Math 121, Sections 01 and 02, Fall 2022

Homework #16

Due **Friday, November 18th** in Gradescope by 11:59 pm ET

Goal: Exploring more of the Relationship between Power Series and functions, including Differentiation and Integration of Power Series. Also *substitution* into a known MacLaurin Series. Also SUMS which are not Geometric.

Find the Series Representation for the following functions using *substitution* and determine the Radius of Convergence R. Simplify.

1. $\dfrac{1}{1+x^2}$
2. $\dfrac{x^2}{x^4+16}$
3. $x^3\cos\left(x^2\right)$
4. $5x^2\sin\left(5x\right)$
5. $\dfrac{d}{dx}\left(x^3\arctan(7x)\right)$
6. $\displaystyle\int x^3\arctan(7x)\ dx$
7. $\dfrac{d}{dx}x^2\ln(1+6x)$
8. $\displaystyle\int x^4e^{-x^3}\ dx$

9\. Find the Series Representation for $f(x) = \dfrac{1}{(1+x)^2}$

Hint: $\dfrac{1}{(1+x)^2} = \dfrac{d}{dx}\left(-\cancelto{PS?}{\dfrac{1}{1+x}}\right) = \ldots$

10\. Prove the Power Series Representation formula for $\arctan x$, as shown in class. Yes, show that $C=0$.

11\. Find Series Representation for $\ln(5-x)$. Solve for C and the Radius R.

Hint: $\ln(5-x) = \displaystyle\int \frac{-1}{5-x}\,dx = \int \frac{-1}{5\left(1-\frac{x}{5}\right)}\,dx = -\frac{1}{5}\int \cancelto{PS?}{\frac{1}{1-\dfrac{x}{5}}}\,dx = \ldots$

12\. Find the MacLaurin Series for $f(x) = e^{-2x}$ using two different methods.

First, using the *Definition* of the MacLaurin Series ("Chart Method").

Second, use Substitution into a known series. Your answers should be in Sigma notation.

Continue to next page

13. You do **not** need to state the Radius. Answers should be in Sigma notation $\sum\limits_{n=0}^{\infty}$ here.

You may use the fact that $\boxed{\sin x = \sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}$ without extra justification.

(a) Use the Definition ("Chart Method") to compute the MacLaurin Series for $F(x) = \cos x$.

(b) Use Differentiation to compute the Series for $F(x) = \cos x$.

(c) Use Integration to compute the Series for $F(x) = \cos x$.
Hints: yes, you should solve for $+C$. yes, C should equal 1. Show why $C = 1$.

Find the Sum of each of the following Series, which do converge.

14. $\sum\limits_{n=0}^{\infty} \frac{7^n}{n!}$

15. $\sum\limits_{n=0}^{\infty} \frac{(-1)^n \, 5^n}{n!}$

16. $\sum\limits_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$

17. $\sum\limits_{n=0}^{\infty} \frac{(-1)^n \, \pi^{2n}}{6^{2n} \, (2n)!}$

18. $\sum\limits_{n=0}^{\infty} \frac{3^n}{5^n \, n!}$

19. $\sum\limits_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

20. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots$

21. $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \ldots$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

7:30–9:00 pm TA Aidee, SMUDD 206

9:00–10:30 pm TA Mia, SMUDD 206

Tuesday: 1:00–4:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Karime, SMUDD 206

9–10:30 pm TA Ali, SMUDD 206

Wednesday: 1:00-3:00 pm

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Ali, SMUDD 206

Thursday: none for Professor

1-2:30 pm TA Mia, SMUDD 205

7:30–9:00 pm TA Aidee, SMUDD 206

9–10:30 pm TA Karime, SMUDD 207

Friday: 12:00–2:00 pm

Pay careful attention to details here.
Manipulating power series requires a balance
of memory and technical skill.