

**Homework #14**

**Due Friday, October 28th** in Gradescope by 11:59 pm ET

**Goal:** Exploring Convergence of Infinite Series. Focus on Absolute and Conditional Convergence...also using the Absolute Convergence Test. Finally... some review problems.

**FIRST:** Read through and understand the following Examples. Determine whether the Series is Absolutely Convergent, Conditionally Convergent, or Divergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 7}{n^7 + 2} \hookrightarrow \text{A.S. } \sum_{n=1}^{\infty} \frac{n^2 + 7}{n^7 + 2} \approx \sum_{n=1}^{\infty} \frac{n^2}{n^7} = \sum_{n=1}^{\infty} \frac{1}{n^5} \text{ Converges } p\text{-series } p = 5 > 1.$$

$$\text{Check: } \lim_{n \rightarrow \infty} \frac{\frac{n^2 + 7}{n^7 + 2}}{\frac{1}{n^5}} = \lim_{n \rightarrow \infty} \frac{n^2 + 7}{n^7 + 2} \cdot \frac{n^5}{1} = \lim_{n \rightarrow \infty} \frac{1 + \frac{7}{n^2}}{1 + \frac{2}{n^7}} = 1 \text{ Finite/Non-zero}$$

The **Absolute Series**  $\sum_{n=1}^{\infty} \frac{n^2 + 7}{n^7 + 2}$  also **Converges** by Limit Comparison Test (LCT).

Finally, the Original Series is Absolutely Convergent (A.C.) (by Definition).

$$\text{Ex: } \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{7n + 3} \hookrightarrow \text{A.S. } \sum_{n=1}^{\infty} \frac{1}{7n + 3} \approx \sum_{n=1}^{\infty} \frac{1}{n} \text{ Divergent Harmonic } p\text{-Series } p = 1$$

$$\text{Check: } \lim_{n \rightarrow \infty} \frac{\frac{1}{7n + 3}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{n}{7n + 3} \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \frac{1}{7 + \frac{3}{n}} = \frac{1}{7} \text{ Finite/Non-zero.}$$

Therefore, the **Absolute Series** also **Diverges** by Limit Comparison Test.

Now, we must examine the original alternating series with the Alternating Series Test.

- Isolate  $b_n = \frac{1}{7n + 3} > 0$

- $\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{1}{7n + 3} = 0$

- Terms Decreasing  $\frac{1}{b_{n+1}} < \frac{1}{b_n}$  because  $b_{n+1} = \frac{1}{7(n+1) + 3} = \frac{1}{7n + 10} < \frac{1}{7n + 3} = b_n$

Therefore, the **Original Series** **Converges** by the Alternating Series Test. Finally, we can conclude the Original Series is Conditionally Convergent (C.C.) (by Definition).

## Now complete the following HW problems

Determine whether the given series is Absolutely Convergent, Conditionally Convergent or Divergent. Number 3 is Ratio Test, but use the AC and CC charts for 1, 2, 4, 5.

1.  $\sum_{n=1}^{\infty} (-1)^n \frac{n^3 + 8}{n^8 + 3}$       2.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n + 1}$       3.  $\sum_{n=1}^{\infty} \frac{(-1)^n (\ln n) (2n)!}{n^n 2^{3n} (n!)}$

4.  $\sum_{n=1}^{\infty} (-1)^n \frac{n + 1}{n^2}$       5.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^3}{n^7 + 2}$

6. Write the statement of the Absolute Convergence Test.

7. Use the Absolute Convergent Test to show that  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 + 1}$  Converges.

8. Use the Absolute Convergent Test to show that  $\sum_{n=1}^{\infty} \frac{(-1)^n \sin^2 n}{n^8 + 2}$  Converges.

Review

9. Show that the Sequence  $\left\{ \left( \frac{n}{n+1} \right)^n \right\}_{n=1}^{\infty}$  Converges to  $\frac{1}{e}$ .

10. Determine if the Series  $\sum_{n=1}^{\infty} \left( \frac{n}{n+1} \right)^n$  Converges or Diverges.

11. Find the Sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^n 5^{2n+1}}{2^{5n-1}}$ .

# REGULAR OFFICE HOURS

**Monday: 12:00–3:00 pm**

7:30–9:00 pm TA Aidee, SMUDD 206

9:00–10:30 pm TA Mia, SMUDD 206

**Tuesday: 1:00–4:00 pm**

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Karime, SMUDD 206

9–10:30 pm TA Ali, SMUDD 206

**Wednesday: 1:00-3:00 pm**

6–7:30 pm TA Admire, SMUDD 206

7:30–9:00 pm TA Ali, SMUDD 206

**Thursday: none for Professor**

1-2:30 pm TA Mia, SMUDD 205

7:30–9:00 pm TA Aidee, SMUDD 206

9–10:30 pm TA Karime, SMUDD 207

**Friday: 12:00–2:00 pm**

This is the end of the material for the Exam 2. Material stops after  
Section 11.6 Absolute Convergence Test and Ratio Test  
and Absolute/Conditional Convergence.